International Journal of Science Education

ISSN: 0950-0693 (Print) 1464-5289 (Online) Journal homepage: http://www.tandfonline.com/loi/tsed20

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Süleyman Turşucu, Jeroen Spandaw, Steven Flipse & Marc J. de Vries

INTERNATIONAL JOURNAL OF SCIENCE EDUCATION, 2017
VOL. 39, NO. 5, 587–604
http://dx.doi.org/10.1080/09500693.2017.1296981

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Teachers' beliefs about improving transfer of algebraic skills from mathematics into physics in senior pre-university education

Süleyman Turşucu [a], Jeroen Spandaw[a,b], Steven Flipse[a] and Marc J. de Vries[a]

[a]Faculty of Applied Sciences, Science Education and Communication, Delft University of Technology, Delft, Netherlands; [b]Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology, Delft, Netherlands

ABSTRACT
Students in senior pre-university education encounter difficulties in the application of mathematics into physics. This paper presents the outcome of an explorative qualitative study of teachers' beliefs about improving the transfer of algebraic skills from mathematics into physics. We interviewed 10 mathematics and 10 physics teachers using a semi-structured questionnaire that was based on an algebraic transfer problem. Almost all teachers acknowledged this transfer problem and considered it to be important. We found a continuum of teachers' beliefs about aspects influencing transfer, including beliefs on improving this transfer. Together with identified improvement aspects about coherent mathematics education, these may help reduce physics teachers' frustrations who spend extra time on re-teaching mathematics. Teachers think that transfer does not happen, because students see both subjects as separate disciplines. Contrary to most physics teachers, most mathematics teachers do not feel the need to collaborate with physics teachers. We found two extreme, opposite beliefs about the transfer of algebraic skills into physics. An intermediate group believes that only an integrated approach can solve the transfer problem. Some of the teachers' beliefs could be organised into a beliefs system. Further research could investigate to which extent such beliefs systems exist and which beliefs these contain.

ARTICLE HISTORY
Received 27 September 2016
Accepted 15 February 2017

KEYWORDS
Teacher beliefs; qualitative research; interview

Introduction

Internationally, educational experts, teachers and policy-makers have stressed the need to integrate – or at least to make connections – between mathematics education and science education (Berlin & White, 2012, 2014; National Research Council, 1996). As part of coherent mathematics education (CME), the United States' NCTM (2000) states that students should be able to transfer mathematics in different contexts outside mathematics. However, research has shown that students encounter mathematical difficulties in

CONTACT Süleyman Turşucu s.tursucu@tudelft Faculty of Applied Sciences, Science Education and Communication, Delft University of Technology, Lorentzweg 1, 2628 CJ, Delft, Netherlands

science subjects, implying a lack of transfer between these subjects (Cui, 2006; Karakok, 2009; Roorda, Goedhart, & Vos, 2014).

The lack of transfer may be related to the mismatch between teachers' beliefs and classroom practice. Indeed, beliefs have a major impact on teacher behaviour (Ernest, 1991). For instance, when physics teachers naïvely (Schoenfeld, 1985) believe that practising a lot in mathematics class leads to better transfer in physics. They do not pay any attention to the mathematics behind physics problems. However, these teachers soon find themselves re-teaching basic mathematics. This may be frustrating (Roorda, 2012) and time-consuming, overshadowing the *science* content that needs to be taught. This paper is concerned with gaining more insight into teachers' beliefs about aforementioned transfer, since this has not been investigated extensively. In particular, we study the application of algebraic skills into physics.

Research aim and questions

This paper aims to report the findings of a qualitative study on mathematics and physics teachers' beliefs about improving transfer of algebraic skills from mathematics into physics. Two research questions will be answered: (A) 'How do mathematics and physics teachers characterise the transfer problem in the case?', and (B) 'What sort of beliefs do mathematics and physics teachers' beliefs have about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education (SPE)?'

These teachers' beliefs can be organised into a belief system (Ernest, 1991), which contains a set of mutually supporting beliefs. In this study, we investigate the individual beliefs, rather than belief systems. As a working definition of 'belief', we used quotes such as 'In my opinion … ', 'I believe … ' and 'I think … ' (Pajares, 1992).

Background

Coherent mathematics education

Mathematics and science are closely connected (Atiyah, 1993). Mathematics provides the tools by which quantitative relationships in science subjects can be represented, modelled, calculated and predicted. Science offers meaning to mathematics by means of rich and relevant contexts in which mathematics can be applied (Dierdorp, Bakker, van Maanen, & Eijkelhof, 2014). Education that has the aim to foster this connection forms the basis of CME and is of vital importance for students (Berlin & White, 2012, 2014). Connecting these subjects is possible through alignment, such as using compatible notations, concept descriptions and pedagogy of mathematical methods. Sufficient attention for the connection between mathematics and physics may improve students' transfer of algebraic skills to physics and strengthen the extent to which students demonstrate CME (Alink, Asselt, & Braber, 2012).

Another way to connect both subjects is through organisation of the learning process in order to achieve a logical learning line across both subjects. In practice, unfortunately, it still happens that certain mathematical concepts are used in physics class *before* they were introduced in mathematics class (Alink et al., 2012).

The CME approach is based on traditional transfer of learning (Singley & Anderson, 1989): application of knowledge learned in a one situation (initial learning) to a new situation. Haskell (2001) states that this is universally accepted as the ultimate aim of teaching. Within this model, the expert (teacher) determines whether transfer occurs or not. However, 'there is little agreement in the scholarly community about the nature of transfer, the extent to which it occurs, and the nature of its underlying mechanisms' (Barnett & Ceci, 2002, p. 612). Hence, there has been a shift from traditional to alternative models, such as actor-oriented transfer. Within this framework, the expert tries to understand the process in which the actor (student) constructs similarities between the initial learning situation and the new situation (Lobato, 2003). The extent to which transfer occurs moves from the experts' to the actors' point of view.

Teachers' naïve beliefs and classroom practice

Beliefs play a critical role in organising knowledge and information and have a major impact on behaviour (Ernest, 1991; Pajares, 1992). As stated earlier, beliefs can be organised into a belief system. This helps individuals to understand and define the world and themselves.

As indicated in Figure 1 by the downward arrows, a teacher's view, i.e. a belief system about the nature of mathematics, provides a basis for the teacher's mental (espoused) models of learning and teaching mathematics. In this study, the espoused model in Figure 1 refers to teachers' naïve beliefs (Schoenfeld, 1985) about improving transfer. Constraints and opportunities provided by the social context of teaching transform these mental models into classroom practice (enacted models). Case studies have shown that there can be a mismatch between a teachers' espoused and enacted model (see Figure 1), indicating a disparity between beliefs and classroom practices (Cooney, 1985). To change unproductive naïve beliefs about transfer, teachers have to be aware of the relation between these beliefs and their classroom practice (Ernest, 1991), reflect about them and reconcile their espoused and enacted beliefs.

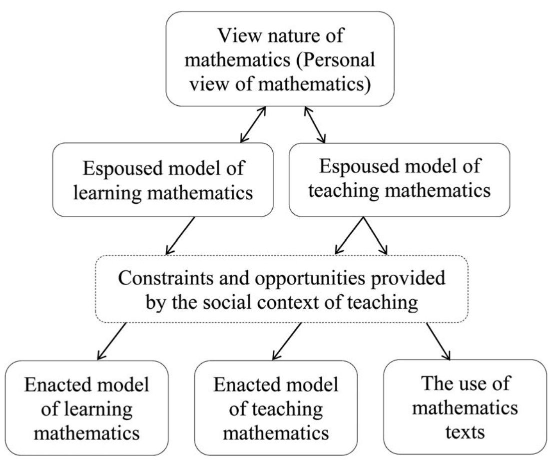

Figure 1. How a teachers' belief system is affected by the social context of teaching (adopted from Ernest, 1991).

The unifying role of mathematics

Naïve beliefs about transfer may be related to beliefs about the unifying role of mathematics (Atiyah, 1993), beliefs about drilling of basic algebraic skills (Wu, 1999), such as adding fractions, formal substitution and completing the square (Drijvers, 2011), and beliefs about automatic transfer of these basis skills to science. However, such beliefs do not take into account conceptual understanding (Kilpatrick, Swafford, & Findell, 2001) and could lead to routine based on tricks. On the other hand, too much focus on conceptual understanding can impede basic skills. We conclude that both basic skills and conceptual understanding have to be taught in an integrated approach (Drijvers, 2011). This may improve transfer.

Whereas most scientists view mathematics as the 'servant of the sciences', many mathematicians consider mathematics to be 'the queen of the sciences' (Atiyah, 1993). They often perceive applied mathematics as inferior to pure mathematics. Some even refuse to discuss applications. Such beliefs may conceivably influence transfer.

CME and transfer in the classroom

CME depends on actors such as teachers, school organisation, curriculum and policy-makers (Schmidt, Wang, & McNight, 2005). Since these actors interact with one another, their involvement makes CME and transfer a rather complex process. As Schmidt, McKnight, and Raizen (1997, p. 92) explain: 'each "actor" pursues his or her own "life" – his or her goals, visions, plans, processes, and efforts to satisfy those to whom he or she is accountable'.

In this study, we restrict ourselves to teachers' beliefs about curricula, textbooks and teachers. Teachers follow the textbooks very closely, and these textbooks are shaped by the curricula (van den Heuvel-Panhuizen & Wijers, 2005; Stein & Smith, 2010).

Classroom actors

Pre-university education (PE) in the Netherlands consists of three junior years (JPE) and three senior years (SPE). In SPE students in physics class have to choose between mathematics A and B. The latter puts much more emphasis on algebra than the former. The content of mathematics and physics is specified in curricula ('Netherlands institute for curriculum development', 2016) and tested in national final central examinations.

According to van Zanten and van den Heuvel-Panhuizen (2014) textbooks mediate between the core goals of education (the intended curriculum) and the actual teaching in classrooms (the implemented curriculum). Hence, textbooks are referred to as the potentially implemented curriculum (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). The limited description of the core goals in the national curricula leave publishers room for different interpretations. Their textbooks are followed very closely by both teachers and students (Stein & Smith, 2010).

Methodology

In this section, we will *first* explain how we collected our data. *Second*, the semi-structured questionnaire used in this study is presented. Finally, we will discuss the methods used to analyse our data.

Table 1. Semi-structured questionnaire, which was based on the transfer problem in the case.

Question number	Rationale
	Case: during a physics lesson a student does not recognize that the physical formula (formula in short) for displacement, $s = (1/2)at^2$, has a similar algebraic structure as the mathematical equation (equation in short), $y = bx^2$. This student is also unable to express t in terms of s. However, earlier that day during mathematics, the student managed to express x in terms of y, implying that besides a lack of recognition, the student is not able to apply algebraic skills from mathematics to physics successfully.
	Now we want the same student to *recognize* that in both cases a similar algebraic structure is used.
1.	*Is this a familiar problem?*
2.	*Do you consider it an important problem?*
3.	*What may be the reason?*
4.	*As a physics (mathematics) teacher, what would you do a about it?*
5.	*What may the mathematics (physics) teacher do about it?*
6.	*What does it mean for the formal physics (mathematics) curriculum?*
7.	*What does it mean for the math- and physics textbooks?*
	We also want this student to be competent in the *application* of algebraic skills from mathematics into physics. In this case, to express t in terms of s: $t = \sqrt{(2s/a)}$.
8.	*Do you consider it an important problem?*
9.	*What may be the reason?*
10.	*As a physics (mathematics) teacher, what would you do a about it?*
11.	*What may the mathematics (physics)teacher do about it?*
12.	*What does it mean for the formal physics (mathematics) curriculum?*
13.	*What does it mean for the math- and physics textbooks?*
14.	*To what extent do you follow textbooks during teaching?*
	In the above-mentioned case, it can be seen that math- and physics are closely related to one another. Teachers appear to have different ideas about their relation.
15.	*How do you see the relation between math- and physics?*
16.	*Do you have any cooperation with your mathematics colleagues?*
17.	*How do you see the optimal cooperation with your mathematics colleagues?*
	Our pre-university physics education is permeated with algebraic problems from mathematics, such as the case above.
18.	*How can the application of algebraic skills from mathematics to physics be improved for solving algebraic problems that occur in our pre-university physics education?*

Data collection

Convenience sampling (Bryman, 2012) was used to gather data from 10 Dutch mathematics and 10 Dutch physics teachers. Each group of respondents consisted of eight male and two female teachers, were qualified to teach in SPE and had at least five years of teaching experience. These numbers are in good agreement with the gender ratio in SPE in the Netherlands: about 15% of mathematics teachers and 5% of physics teachers are female (Mullis, Martin, Kennedy, Trong, & Sainsbury, 2009). The respondents were interviewed by means of a questionnaire and gave consent to reporting. Each interview was conducted privately in an appropriate, silent place chosen by the teacher and took 30–45 min. Afterwards, it was transcribed *ad verbatim* for analysis. For the teachers' names, we used pseudonyms.

Semi-structured questionnaire

In order to investigate research questions A and B, we used a semi-structured questionnaire (Bryman, 2012) that was based on one specific *case* about a transfer problem. The questions were based on this case. For the case and the questionnaire, see Table 1.

Data analysis

We used open coding (Bryman, 2012; Saldaña, 2013) to label each fragment of the transcripts, giving us a short description of teachers' beliefs about both research questions. This process resulted for each of the 20 interviews in a set of labels identifying teachers' beliefs.

Next, we used axial coding, consisting of two steps (Bryman, 2012; Saldaña, 2013). In the first step, labels with roughly the same content were grouped together, leading to a grouping of the labels. Each group of labels was summarised as a subtheme. A subtheme had to contain at least three different beliefs of at least three different teachers. Otherwise, it was marked as an outlier. In the second step, we grouped the 28 subthemes into 9 core themes (see Table 2). Thus, we obtained one common code tree for all 20 teachers. This tree is a hierarchical structure consisting of core themes as main branches. These core themes then branch out into smaller branches, called subthemes. The next and finest level of the hierarchy, the leaves of the tree, are the underlying teachers' beliefs about CME and aspects influencing students' transfer of algebra to physics.

Table 2. Teachers' beliefs about aspects influencing students' transfer and aspects about CME.

Core theme/subtheme	Mathematics teachers	Physics teachers
1. Coherence	126	135
1.1 Alignment	2/1[a]	10/6
1.2 Collaboration and cooperation	85/10	75/10
1.3 Ideal collaboration and cooperation	39/10	50/10
2. Curriculum	65	86
2.1 Curriculum (general)	25/9	10/7
2.2 Mathematics curriculum	23/10	31/10
2.3 Physics curriculum	17/10	45/10
3. Education	7	26
3.1 Junior pre-university education	7/5	26/7
4. Pedagogy of algebra	82	72
4.1 Algebraic skills	40/10	26/7
4.2 Algebraic techniques	7/4	8/5
4.3 Practice (general)	21/9	30/9
4.4 Practice within mathematics	9/5	3/3
4.5 Practice within physics	5/3	5/3
5. Relation between scientific subjects	87	52
5.1 Mathematics and physics	27/10	15/10
5.2 Mathematics within physics	35/10	23/10
5.3 Physics within mathematics	25/10	14/10
6. School subjects	30	20
5.1 Mathematics	19/7	13/6
5.2 Physics	11/6	7/4
7. Teacher	193	112
6.1 Mathematics teacher	97/10	48/10
6.2 Physics teacher	96/10	64/10
8. The use of textbooks	143	139
8.1 Following textbooks	31/10	43/10
8.2 Mathematics textbook	66/10	31/10
8.3 Physics textbook	37/10	45/10
8.4 Textbook general	9/5	20/7
9. Transfer	144	89
9.1 Activating prior knowledge	8/5	10/4
9.2 Affordances (specific)	34/10	8/5
9.3 Constructing relations (general constraints)	27/10	23/9
9.4 Constructing relations (specific constraints)	75/10	48/10
9.5 Focus on students	1/1[a]	1/1[a]

[a]This subtheme turned out to be an outlier.

To enhance reliability of our results, the whole process of open and axial coding was independently carried out by another researcher. The two common code trees overlapped for approximately 80% on each of the three levels (labels, subthemes and core themes). The two researchers then discussed the remaining 20%. After some adjustments in these parts of the tree, this led to consensus among the two researchers about the common code tree. Finally, the whole process was double checked by the second/third author.

Results

First, we present our results for research question A, then for research question B.

Research question A: characterising the case

Research question A is related to the case and to questions 1, 2 and 8 of the questionnaire in Table 1. As to question 1, 9 out of 10 mathematics teachers, and 8 out of 10 physics teachers acknowledged the case. As to question 2, nine mathematics and nine physics teachers considered it important. For question 8, we found that nine mathematics and nine physics teachers considered it important that students are competent at the transfer of algebraic skills from mathematics into physics.

Research question B: common code tree

We found a continuum of teachers' beliefs (approximately 1300 beliefs) which can be organised in 9 core themes and their 28 subthemes. For example, the core theme 'School subjects' contains the subthemes 'Mathematics' and 'Physics'. The number '30' next to 'School subjects' is the total number of beliefs about this core theme uttered by the mathematics teachers. The numbers '11/6' next to the subtheme '6.2 Physics' in the same column mean that among these 30 beliefs 11 belonged to this subtheme and they were uttered by 6 teachers. We found three outliers: the subtheme 'Alignment' for mathematics teachers and the subtheme 'Focus on students' for both teacher groups.

Results interpretation

In this section, we will first interpret the results regarding research questions A and B. Next, we will discuss three teacher groups' beliefs about improving transfer of algebraic skills to physics. Finally, we will discuss limitations and recommendations.

The quotations below are taken from the interviews. For stylistic reasons, we use the words 'believe' and 'think' interchangeably to describe teachers' beliefs. The word 'collaboration' refers to activities in which teachers work together, such as designing teaching materials. The word 'cooperation' refers to conversations without such activities.

Our analysis below shows some inconsistencies within the set of beliefs of many interviewees. Indeed, during the second half of the interview many teachers expressed opinions contradicting their own opinions during the first half of the interview. For example, most mathematics and physics teachers first expressed the opinion that extensive algebraic practice in math class alone should solve the transfer problem, but later the same teachers said that algebraic practice is also needed in physics class.

Research question A: insight in the case

Most of the interviewed teachers acknowledged the case of Table 1. This justifies our interviews with both mathematics and physics teachers. It is remarkable that the mathematics teachers acknowledged the case, even though they do not encounter this problem in their own classroom. This may imply that mathematics teachers discuss this problem with physics teachers. Mathematics teachers think that the transfer problem occurs especially in the first year of SPE, rarely in the next years. This seems reasonable, since the level of algebraic skills needed in physics increases substantially in the transition from JPE to SPE.

Most of the physics teachers think that the well-performing students in mathematics B do not encounter any transfer problems at all. This belief is supported by the fact that mathematics B puts a much stronger emphasis on algebra than mathematics A.

Regarding questions 2 and 8, most physics teachers believe that in recent years, the transfer of mathematics to physics has become more important, because this transfer plays a larger role in last year's central physics exams. They think that the relation (they often used the word 'link') between mathematics and physics has to be emphasised more strongly. Most of the mathematics teachers, however, used the word 'application'; they mention the importance of applying mathematics to another subject such as physics.

Research question B: core themes

Below, we discuss the subthemes for each core theme (see Table 2).

Core theme 1: coherence

The subtheme 'Alignment' turned out to be an outlier for mathematics teachers. Most of the physics and mathematics teachers mention the need to align the learning lines in mathematics and physics using the textbooks. This connection is of key importance: it may improve students' transfer of mathematics into physics and also strengthen the extent to which they experience coherence between these subjects (Berlin & White, 2012, 2014).

As for the subtheme 'Collaboration and cooperation', most of the physics teachers say they are willing to collaborate, but they strongly believe that mathematic teachers do not feel the need for collaboration. As one of them said, 'It is difficult to communicate with mathematics teachers.' Consequently, there is little interaction between mathematics and physics teachers. If there is any interaction at all, this consists of individual efforts on a small scale during informal meetings. Indeed, our data indicate the existence of *two* types of mathematics teachers. The first type, which represents the majority, does not feel the need to collaborate with physics teachers, supporting physics teachers' beliefs. They think that 'They [physics teachers] have a problem, and they have to find us.' The second type does collaborate with physics teachers. These mathematics teachers also feel the need to align the content of mathematics and physics subjects across time.

The next subtheme 'Ideal collaboration and cooperation' assumes the absence of constraints (see Figure 1). Most of the mathematics teachers believe that more collaboration with their colleagues from physics would be desirable in this ideal situation. The difference between this ideal (espoused) beliefs of mathematics teachers and their lack of collaboration with physics teachers (enacted beliefs) can be caused by constraints (see Figure 1). Indeed, they often mentioned huge workload as an impeding factor. The physics teachers believe that an ideal collaboration would result in alignment of notations, equations, formulas and algebraic techniques in both subjects.

Core theme 2: curriculum

Both mathematics and physics teachers use the words 'connection' and 'integration' interchangeably to indicate CME. Concerning the subtheme 'Curriculum (general)' most of the physics teachers think that there is the need to integrate or at least make connections between the mathematics and physics curriculum. Although they do not explicate what this integration or connection should look like, they believe that these should be visible through the content standards, probably because they observe which algebraic skills their students lack in physics class. In contrast, most of the mathematics teachers think that such integration or connection is not needed. Presumably, they are unaware of the type of mathematical skills that students lack during physics lessons.

Regarding the subtheme 'Mathematics curriculum', most of the mathematics and physics teachers think that the content standards should include physics contexts in which algebraic skills are involved. For instance, manipulating formulas and solving for a variable. A small number of these teachers (including some math teachers!) state that they are unaware of the content of the mathematics curriculum. Indeed, most teachers rely on textbooks as a substitute for the curriculum. They trust that these textbooks represent this curriculum accurately, and they follow these books very faithfully (Stein & Smith, 2010; van Zanten & van den Heuvel-Panhuizen, 2014). Most of the mathematics teachers desire the incorporation – in the curriculum or in the textbooks; for many teachers that is the same thing – of a content standard about recognising the algebraic structure of formulas and equations in physics.

For the subtheme 'Physics curriculum', most of the mathematics and physics teachers would like to see an emphasis on algebraic skills, e.g. manipulating formulas and solving for variables in the physics curriculum. Some mathematics teachers give quite explicit suggestions about what is needed in the physics curriculum. This result is quite remarkable, since most mathematics teachers do not feel the need to integrate both curricula. Physics teachers also wish for a content standard about recognition of the algebraic structure of formulas and equations in physics. However, a small number of physics teachers seem to be satisfied with the actual physics curriculum: 'There is no need to add anything.'

Core theme 3: education

Most mathematics and physics teachers think that in the last year of JPE, there is a lack of emphasis on algebraic skills in mathematics lessons. Physics teachers mention that they observe this lack above mainly in the first year of SPE. As mentioned above, this belief is shared by most mathematics teachers.

Core theme 4: pedagogy of algebra

Concerning the subtheme 'Algebraic skills' most of the physics teachers again mention that the lack of sufficient algebraic skills to tackle transfer problems mainly occurs in the first year of SPE. This result is in agreement with the subsections 'Discussion on characterising the case' and the core theme 'Education'. Mathematics teachers think that more practice with algebraic skills will improve transfer.

The subtheme 'Algebraic techniques' concerns mathematical tools used to solve algebraic problems such as cross multiplication and cover-up method (Drijvers, 2011). Both mathematics and physics teachers think that there is a mismatch between algebraic techniques learned in mathematics and physics. They think that more alignment between these algebraic techniques is needed.

As to the subtheme 'Practice (general)', both groups think that the lack of practice with transfer problems analogous to the case impedes the transfer of algebraic skills to physics. They believe that more practice in both physics and mathematics is required to improve transfer. This is illustrated by a quote from a physics teacher, referring to both subjects: 'In physics class students should practice with formulas analogous to $s = (1/2)at^2$ and in math class with equations analogous to $y = bx^2$. This will help students to solve this transfer problem.' Similar statements were made by many other mathematics and physics teachers.

Regarding the subtheme 'Practice within mathematics', most of the mathematics and physics teachers think that extensive practice in math class with algebraic skills is both necessary and sufficient. This is illustrated by the quote, 'They need lots of practice during mathematics classrooms. Then, application into physics will happen automatically.' A small number of mathematics and physics teachers think that in math classes more practice with transfer problems analogous to the case is needed.

As for 'Practice within physics', most mathematics and physics teachers believe that in physics classes, more practice with physics problems involving algebraic skills is needed. This contradicts their previous statement about the alleged sufficiency of practice in math class and automatic transfer. A small number of mathematics and physics teachers suggested activation of prior mathematical knowledge by starting with the mathematics problem in the case (see Table 1), followed by algebra problems in physics.

Summarising, both teacher groups put a strong emphasis on practice with transfer problems similar to the case and believe that this would improve students' transfer. This belief may be regarded as naïve and can be associated with the idea of basic skills first (Wu, 1999). However, there is no single teacher who mentions and relates this matter to conceptual understanding in both activities. This result is important, because it might indicate that these teachers overlook a serious risk: putting too much emphasis on basic skills could push conceptual understanding of the underlying mathematics to the background (Drijvers, 2011; Kilpatrick et al., 2001) and could impede transfer of algebra to science. Hence, teachers who develop common learning strategies aiming at transfer should take into account both basic skills and conceptual understanding. Note that this result may also partly explain the lack of transfer in earlier studies (Cui, 2006; Karakok, 2009; Karam, 2014; Roorda, 2012).

Core theme 5: relation between scientific subjects

Concerning the subtheme 'Mathematics and physics', most of the mathematics and physics teachers think that mathematics and physics are two inextricably intertwined subjects. Only a small number of mathematics and physics teachers think that both subjects should be regarded as separate disciplines. Some mathematics teachers view mathematics as the 'Queen of all sciences' (Atiyah, 1993): mathematics should remain pure, because application of mathematics would degrade it.

> As to the subtheme 'Physics within mathematics', most of the physics teachers view mathematics as the 'Servant of science' (Atiyah, 1993).

> As to the role of 'Mathematics within physics', most teachers in both groups mentioned 'aid' and 'tool'. Some of the math teachers even mentioned 'mathematics serves physics' (cf. Atiyah, 1993).

Before we conducted the interviews with teachers, we hypothesised that teachers who view mathematics as the 'Queen of all sciences' would not feel the need to bother about transfer. However, analysis of the data shows the opposite: these teachers *did* make suggestions about tackling transfer. This result seems to indicate that they are aware of the importance of applying mathematics in physics, even though they have purist views about mathematics.

Core theme 6: school subjects

Concerning the subtheme 'Mathematics', most of the physics teachers believe that physics should provide good contexts for math class. Examples from physics in math class makes mathematics more understandable and offer new insights to the students. This result matches with the view expressed in Alink et al. (2012) and Berlin and White (2012, 2014), who write that science contexts offer meaning to mathematics in which it can be applied, and they contribute to students experiencing CME. Unfortunately, the beliefs of the interviewed mathematics teachers were too fragmented to draw conclusions.

This fragmentation of mathematics teachers' beliefs also holds for the subtheme 'Physics'. Still, a small number of them think that the mathematics used in physics class should be restricted to mathematics A, since some students in physics class do not study mathematics B (see the first paragraph of subsection 'Classroom Actors' for information about mathematics A and B). Physics teachers share this belief. They add mathematics A students encounter more difficulties with algebraic problems in physics class than mathematics B students. This belief may indicate that physics teachers are aware of the fact that some of their students have less training in algebraic skills because they do not study mathematics B, although none of the physics teachers explicated this.

Core theme 7: teacher

Concerning the subtheme 'Mathematics teachers', most of the mathematics and physics teachers agree that math teachers should incorporate more physics context in their lessons. Furthermore, they should emphasise the close relationship between mathematics and physics. Most of the physics teachers think that mathematics teachers should include exercises similar to the case. A small number of physics teachers mention that mathematics teachers should be competent in the mathematics content. They should use other variables than x and y. Another small number of physics teachers mention that mathematics teachers should be acquainted with the physics curriculum, but they do not specify to what extent. A small number of mathematics and physics teachers think that mathematics teachers should stick to pure mathematics: ‘Mathematics should avoid all forms of physics.’ Unlike the physics teachers, the math teachers in this group view mathematics as the ‘Queen of all sciences’ (Atiyah, 1993). However, these math teachers do make suggestions on how to improve transfer. Another small number of mathematics teachers mention exercises similar to the case and use other variables than just x and y.

Regarding the subtheme ‘Physics teacher’, most of the physics teachers mention the desirability of teacher-centred practice of physics problems which involve algebra in physics class. They also believe that showing similarities between different equations and formulas is beneficial. They emphasise that students should practice with exercises similar or analogous to the case. Prior mathematical knowledge that is related to the mathematics involved in physics problems should be activated. Physics teachers should use x and y as well as the conventional quantities in physics. This can be regarded as an extension of activation of prior mathematical knowledge. A small number of physics teachers use their privately developed teaching materials to train students’ mathematical skills: ‘Students’ performance on algebraic skills were bad. I became frustrated and developed my own teaching material.’ This may be due to the absence of sufficient attention on algebraic skills in the current physics methods. Another small number of physics teachers mention the lack of time to focus on algebraic skills in physics problems. This shortage of time is often observed in schools in the Netherlands and most probably related to physics teachers’ busy daily routine (Alink et al., 2012). As opposed to physics teachers, most of the mathematics teachers put a stronger emphasis on activating prior mathematical knowledge. They believe that they should relate physical quantities to the variables x and y used in mathematics, write down a formula from physics next to the corresponding mathematical equation on the blackboard. They also mention practice with exercises similar to the case and explanation about the close relationship between mathematics and physics. A small number of mathematics teachers think that physics teachers explicating transfer problems in physics lessons to mathematics teachers may also help to reduce these transfer problems.

We conclude there are many beliefs about what mathematics and physics teachers should do to deal with the problem of transfer. According to Davison, Miller, and Metheny (1995), these beliefs help students to, ‘ … explore the connections between mathematics and science and begin to see the relevancy of mathematics in the reality of science and vice versa’ (p. 228). Sufficient attention for these connections may help to improve transfer and enhance students’ experience of CME (Alink et al., 2012; Berlin & White, 2012, 2014).

Core theme 8: the use of textbooks

As to the subtheme ‘Following textbooks’, all interviewees mention that they are highly textbook-driven. This is in line with earlier research (Stein & Smith, 2010; van den Heuvel-Panhuizen & Wijers, 2005).

The beliefs belonging to the subtheme ‘Mathematics textbook’ indicate the existence of two main types of mathematics teachers. The first type claims to be satisfied with the use of contexts in mathematics textbooks, e.g. ‘There is enough context [in the mathematics method].’ The second type would like to see more context in mathematics textbooks. They also frequently mention the lack of sufficiently many formulas and physics exercises. Most of the physics teachers advocate the inclusion of more physics context in mathematics textbooks, such as formulas, exercises and algebraic skills needed to solve physics problems. A small number of physics teachers, however, disagree on this point.

Concerning the subtheme 'Physics textbook', the majority of the interviewed physics teachers think that physics textbooks need introduction paragraphs containing prior mathematical knowledge about the physics content that will be treated. They believe that some physics textbooks do this adequately, whereas others do not. Only two physics teachers were satisfied with the actual content of physics textbooks. Most of the mathematics teachers strongly belief that activating prior mathematical knowledge is of key importance in the approach of tackling transfer problems.

Regarding the subtheme 'Textbook (general)', the teachers use the words 'connection' and 'integration' interchangeably to indicate connection in terms of alignment between mathematics and physics textbooks. Most of the physics teachers would like to see this connection in textbook series. This should be made possible through exercises analogous to the case, equations and corresponding formulas that are treated together, or alignment of algebraic techniques in both textbooks. They also mention alignment of textbooks on a general level, without making any concrete suggestions. Mathematics teachers' beliefs are split: while one part thinks there is the need to connect both textbooks, the other part shows very little enthusiasm. Supporters of connection would like to see two separate textbooks, with the physics textbooks referring to mathematics textbooks, and vice versa. Data show that teachers who advocate such connection do not speak about one single integrated textbook, but two separate textbooks making connections to one another through content.

Still, aiming at CME through connections between the content of mathematics and physics textbooks is a rather complex process, which depends on good collaboration between other actors than just textbook publishers, such as policy-makers (Schmidt et al., 2005).

Core theme 9: transfer

Although mathematics and physics teachers' beliefs about the subtheme 'Activating prior knowledge' are very fragmented, they all mention the importance of activation of prior mathematical knowledge in physics class.

For the subtheme 'Constructing relations (affordances)', most of the mathematics and physics teachers believe that the transfer problem in the case may be overcome when students recognise similarities in the algebraic structures of equations and formulas. They also think that physics teachers have to relate quantities of formulas more often to the variables x and y from mathematics. This can be interpreted as activation of prior mathematical knowledge.

Regarding the subtheme 'Constructing relations (general constraints)', most of the mathematics and physics teachers believe that the main reason for students facing difficulties with transfer problems analogous to the case is because students see both subjects as two entirely different subjects, two separate worlds: 'Students think that they have entered an entirely new subject when just having left math and entered the physics classroom.' This is in line with the literature where mathematics and science, in particular physics are seen as two separate subjects (Cui, 2006; Karakok, 2009; Roorda, 2012).

As to the subtheme 'Constructing relations (specific constraints)', most of the mathematics and physics teachers think that the variable names x and y are often used in mathematics, but impede transfer. A small number of mathematics and physics teachers also think that this transfer is impeded when students rely too much on mathematical tricks, such as the equation triangle. These teachers seem to be aware of the necessity of conceptual understanding (Kilpatrick et al., 2001). Furthermore, a small number of mathematics teachers think that the absence of automation in solving transfer problems impedes transfer too. However, from data it is not clear what they exactly mean with automation.

Three approaches to transfer

Most of the interviewees belong to one of the following three groups. The first and largest group believes that the transfer problem should by intensive algebraic practice in math class. Then, they claim, transfer of algebraic skills into physics happens automatically. The second and smallest group believes that the transfer problem should be solved by practising algebraic physics problems in physics class. The third group lies between these opposite views. These teachers believe that the transfer problem can only be solved by comprehensive algebraic practice in both mathematics and physics class. For example, algebra problems in math class should use contexts and notations from physics, and physics teachers should activate prior mathematical knowledge. Both physics and mathematics teachers should emphasise the connections between their subjects.

The believes in the first group are linked to the unifying role of mathematics (Atiyah, 1993) and can be interpreted as prioritising basic skills (Wu, 1999). The second group's believes can be described as reinventing the same mathematical wheel in different physics contexts. Presumably, the same wheels also have to be reinvented in other subjects using algebra such as chemistry and economics. Although this approach does not concern mathematics lessons, it still can be viewed as prioritising basic skills, but in science context.

We conclude that the first and second groups ignore the development of conceptual understanding in their teaching (Kilpatrick et al., 2001), meaning that the understanding of the underlying mathematics could be pushed into the background (Drijvers, 2011). However, for optimal transfer conditions there must be a focus on both basic skills and conceptual understanding (Kilpatrick et al., 2001; Roorda et al., 2014). Thus, teachers' beliefs prioritising basic skills can be seen as naïve (Schoenfeld, 1985). Teachers who transform such beliefs (espoused model) into teaching practice (enacted model) may be confronted with a great disparity between beliefs and what they observe in the classroom (see Figure 1), i.e. a lack of transfer. Teachers should be aware of the existence of such beliefs, reflect on these and reconcile with their classroom practice. Without reflectivity, teachers are often observed to adopt similar practices in the classroom (Ernest, 1991; Pajares, 1992). As a consequence, students may lack of transfer in physics.

The third group's beliefs about transfer are most constructive, since they take into account an integrated approach. It contributes to students experiencing coherence across both subjects. However, not all teachers in this group pay sufficient attention to conceptual understanding.

Limitations of this study and recommendations

For this study, we interviewed teachers from SPE from different schools in the Netherlands within a radius of approximately 50 km. These teachers were randomly chosen and had varying years of teaching experience, i.e. ranging from 5 to 40 years. In terms of diversity in teachers' beliefs, we did not observe much change after a total of eight interviews including four mathematics and four physics teachers, indicating saturation for both teacher groups.

On the basis of the results above, we expect this study to be generalisable for mathematics and physics teachers teaching in SPE in the Netherlands and also for those who teach in senior general secondary education. However, we do not expect that this holds for preparatory vocational secondary education, because the mathematical skills needed in physics are fundamentally different from those in senior general secondary education and SPE. This can lead to different teachers' beliefs about transfer.

We think that these teachers may be representative for Dutch teachers who teach at SPE. First of all, they were all qualified to teach in this education sector. Second, mathematics and physics education in the Netherlands is centralised through curricula, shaping to a very large extent the content of textbooks and teachers who quite strictly follow these textbooks (van den Heuvel-Panhuizen & Wijers, 2005; Stein & Smith, 2010). Thus, to a great extent Dutch teachers' beliefs about transfer are influenced by textbooks. However, we do not expect much difference in the content of textbook series, implying that teachers' beliefs above would not differ significantly from each other. In many countries, however, the combination of such centralised curricula shaping textbooks with textbook-driven teachers does not exist (Valverde et al., 2002), implying that our results are not generalisable to these countries.

Three of our subthemes were removed, because these did not consist of at least three different beliefs mentioned by at least three different teachers. This criterion is slightly arbitrary. Data reduction is associated with grasping the essence and leaving out less important details. Outliers may contain important information about missing teachers' beliefs as the subtheme 'Focus on students' showed. Indeed, only two teachers mentioned the focus on students, meaning that almost all teachers approach transfer traditionally (Singley & Anderson, 1989). These two teachers seemed to adopt an alternative approach in which they tried to understand transfer as constructed by the student, i.e. from the students' point of view, and not from a teachers' perspective (Lobato, 2003). Hence, in terms of improving students' transfer of algebraic skills into physics, we recommend to further investigate the focus on this matter.

Referring to Figure 1, we recommend examining mathematics teachers who had purist beliefs (espoused beliefs), but nevertheless made suggestions about improving transfer. How do they deal with transfer problems in the classroom (enacted beliefs) if they have such purist beliefs?

Conclusion

Regarding research question A 'How do mathematics and physics teachers characterise the transfer problem in the case?' we found that nearly all mathematics and physics teachers acknowledged the case presented to them and considered it important that students are competent at the transfer of algebraic skills from mathematics into physics.

They think that transfer problems occur especially in the first year of SPE. To answer research question B 'What sort of beliefs do mathematics and physics teachers have about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education (SPE)?' we used open and axial coding to analyse the interviews and found one common code tree for both teacher groups, including nine core themes: coherence, curriculum, education, pedagogy of algebra, relation between scientific subjects, school subjects, teacher, the use of textbooks and transfer (see Table 2). These core themes contained a continuum of teachers' beliefs about aspects influencing students' transfer above, including beliefs on how to improve this transfer, and aspects about CME, including aspects that may enhance students' experience of CME (NCTM, 2000). When solved, these aspects may help reduce science teachers' frustrations, who spend extra time on repeating mathematics in science classes.

We have seen that most of the teachers believe that transfer does not happen because students see both subjects as separate disciplines. This is in line with earlier findings (Cui, 2006; Karakok, 2009; Roorda, 2012).

Contrary to physics teachers, most of the mathematics teachers mentioned that they do not feel the need to collaborate and cooperate with physics teachers. This impedes the development of common teaching strategies to tackle transfer problems.

With regard to their views about improving transfer, most interviewees fit into one of the following groups. The first and largest group think that the transfer problem is solved by intensive practice in math class. The second and smallest group believes the opposite: the transfer problem should be tackled by algebraic problems in physics class. Finally, the intermediate group believes in comprehensive algebraic practice in both mathematics and physics class. Conceptual understanding is ignored by all teachers from the first two, extreme groups and by some teachers of the intermediate group.

Some of the teachers' beliefs can be organised into a belief system (Ernest, 1991), i.e. into a set of mutually supporting beliefs about transfer. Further research should investigate to which extent this is the case and which beliefs they contain.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by Nederlandse Organisatie voor Wetenschappelijk Onderzoek.

Notes on contributors

Süleyman Turşucu is a physics teacher at Wolfert TTO in Rotterdam and a PhD student at Delft University of Technology (Netherlands).

Jeroen Spandaw is associate professor mathematics and mathematics education at Delft University of Technology.

Steven Flipse is assistant professor science communication at Delft University of Technology and the founding director of S.Flipse Training & Consultancy, a consultancy firm that supports organisations in responsible (innovation) project management.

Marc J. de Vries is professor of science education at Delft University of Technology, affiliate professor of philosophy and pedagogy of technology and engineering education at KTH Royal Institute of Technology in Stockholm (Sweden). He is also the editor in chief of *The International Journal for Technology and Design Education*.

ORCID

Süleyman Turşucu http://orcid.org/0000-0002-5308-8793

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