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Learning Visualization Strategies: A qualitative investigation

Daniel Halpern[a*], Kyong Eun Oh[b], Marilyn Tremaine[c], James Chiang[c], Karen Bemis[d] and Deborah Silver[c]
[a]*School of Communications, Pontificia Universidad Catolica de Chile, Santiago, Chile;* [b]*Graduate School of Library and Information Science, Simmons College, Boston, MA, USA;* [c]*Electrical and Computer Engineering, Rutgers, the State University of New Jersey, Piscataway, NJ, USA;* [d]*Institute of Marine and Costal Sciences, School of Environmental and Biological Sciences, Rutgers, the State University of New Jersey, New Brunswick, NJ, USA*

The following study investigates the range of strategies individuals develop to infer and interpret cross-sections of three-dimensional objects. We focus on the identification of mental representations and problem-solving processes made by 11 individuals with the goal of building training applications that integrate the strategies developed by the participants in our study. Our results suggest that although spatial transformation and perspective-taking techniques are useful for visualizing cross-section problems, these visual processes are augmented by analytical thinking. Further, our study shows that participants employ general analytic strategies for extended periods which evolve through practice into a set of progressively more expert strategies. Theoretical implications are discussed and five main findings are recommended for integration into the design of education software that facilitates visual learning and comprehension.

Keywords: *Visual learning; 3D visualization; Spatial transformation; Spatial problem-solving; Perspective-taking; Visual comprehension*

Visualizations have traditionally been used in science as an additional resource for teachers to help students to understand abstract concepts that are difficult to describe or phenomena that cannot be observed directly (Buckley, 2000). Science 'visualizations' present data in novel ways to foster student comprehension (Gilbert, 2007). As Mayer and Gallini (1990) suggest, despite the bias toward verbal over visual forms of

*Corresponding author. School of Communications, Pontificia Universidad Catolica de Chile, Alameda 340, Santiago, Chile. Emails: dmhalper@uc.cl, halperndaniel@gmail.com

instruction, research has found a tremendous potential in visually based instruction to facilitate students' understanding of scientific material. In fact, even 'non-spatial' problems are more effectively addressed by the insertion of visualizations (Wu & Shah, 2004), and there is a consensus among scholars that student achievement in science is generally supported by direct access to multimedia modes of representation (Uttal & Doherty, 2008). In chemistry for instance, by visualizing sketched structures, symbols, arrows and equations, students can actually 'see' the chemical process (Kozma, Chin, Russell, & Marx, 2000). These chemical representations allow learners to think visually and convey information efficiently through a form of visual display (Wu & Shah, 2004).

Rapp and Kurby (2008) explain that by making visually explicit complex processes, these visualizations facilitate students' ability to make new links between concepts they already know with the science being explained: if students know, for instance, that particular colors are associated with specific temperatures (e.g. red represents hot), they can use the information to quickly understand the meaning of color cues in a depth-related water, earth or air temperature diagram. However, research has consistently found that students exhibit difficulties in interpreting cross-section visualizations and anatomical structures (Russell-Gebbett, 1985), one of the most utilized representations in science. Treagust, Chittleborough, and Mamiala (2003) report that learners experience difficulties in connecting properties of a molecule with its formula, whereas in chemistry and geology students have difficulties in interpreting symbolic representations and transforming them into three-dimensional (3D) structures (Furió & Calatayud, 1996).

Similarly, Kali and Orion (1996) studied the 3D spatial abilities of geology students by measuring their capacity to comprehend the 3D structure of folded sedimentary rocks from surface structure visualizations. They found that students had different Visual Penetrative Ability, which is 'the ability mentally to penetrate the image of a structure' (Kali & Orion, 1996, p. 369) when solving geology problems. Some of them could not associate the surface information of the geological structures with their interior. These studies indicate that students, paradoxically, have problems in comprehending the very images designed to facilitate their understanding, and thus, illustrate the importance to consider not only how visualizations are designed, but also the way they are interpreted (Pozzer-Ardenghi & Roth, 2005).

Not surprisingly, research shows that spatial intelligence plays a central role in learning through use of visualizations. These are the capabilities to perceive the visual world accurately, to develop transformations upon individual's visual experience, even in the absence of physical stimulation (Gilbert, 2007). Further, several studies have demonstrated that spatial intelligence can be developed through training, instruction in academic settings (Wright, Thompson, Ganis, Newcombe, & Kosslyn, 2008), and even by playing video games (Feng, Spence, & Pratt, 2007). Piburn et al. (2005) designed an experiment to assess the role of spatial ability in learning geology using multimedia instructional modules that used visualizations. They found that subjects improved their spatial visualization and geospatial examination, demonstrating that spatial ability is improved through instruction. Nevertheless, little research has been done on how individuals learn from similar sets of problems and then develop their own strategies to solve visualization tasks.

For this purpose, a test using 3D block visualization problems was designed to gain a deeper understanding about the learning process and strategies developed by individuals in order to visualize the internal structure of 3D visualizations. This technique, which attempts to provide the visual experience of the real world to users, has become a relevant tool for learning purposes in geology, where students have to understand the different layers underneath geological structures (Kali & Orion, 1996). More specifically, cross-sections of 3D objects have been frequently used in textbooks to help students gain a better understanding of the underlying object. Loosely defined by Russel-Gebbett (1985) as the flat surface that comes out when a 3D object is cut by a 2D plane, these types of representations that require observers to mentally visualize the underlying features of the structure depicted have become the standard practice in many disciplines of science. Research in this area has focused on how accurately students visualize the internal properties of the 3D diagrams by centering on problem-solving tasks (Kali & Orion, 1996). Figure 1 shows different cross-section problems in which subjects were asked to imagine how the layer that the slice cuts looks and then to draw the resulting cross-sectional slice.

Because particular attention was paid to the reasoning processes of participants as they tried to overcome the difficulties of the diagrams that prevented their

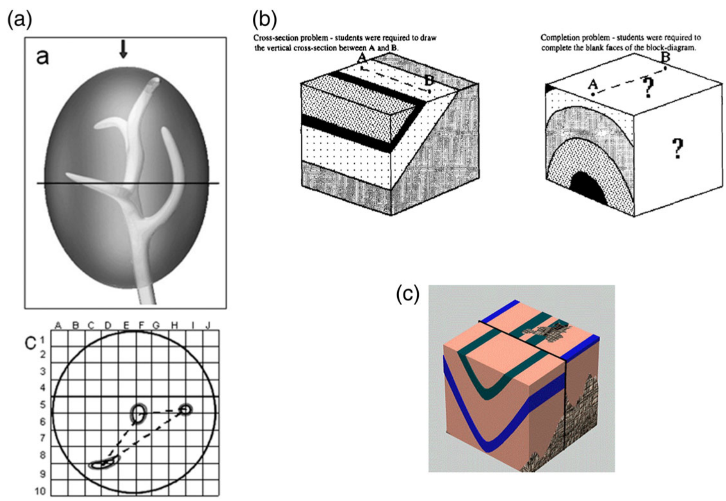


Figure 1 (a) Presents a stimulus of 3D object showing the cutting plane and viewing-direction arrow for one trial, and below the arrow view for the trial shown (Keehner et al., 2008); (b) Shows one of the tasks by Kali and Orion (1996) to determine the level of visual penetration ability in students, and (c) presents an integrative quiz question asking students to place the events (faulting versus intrusion) in the order they must have happened (Piburn et al., 2005) comprehension, we believe the conclusions obtained in the analysis may be useful to integrate into the design of intelligent software that facilitates internal visual learning and comprehension processes.

1. Literature Review

Research has suggested that spatial intelligence is central to careers that required high spatial reasoning levels such as geology (Kali & Orion, 1996). Consequently, discovering how to increase one's level of spatial abilities is relevant for basic education. However, spatial abilities are not typically taught as part of core education practices and thus, are rarely considered as influential on students' academic performance (Webb, Lubinski, & Benbow, 2007). Auspiciously, numerous studies suggest that performance on spatial tasks can be improved through training (Tzuriel & Egozi, 2010) and that practice in different domains may serve to improve other spatial abilities (Terlecki, Newcombe, & Little, 2008).

1.1. *Learning Visualizations Strategies: An holistic approach*

Since 1920, factor analytic studies have explored how subjects employ different strategies to solve spatial tasks, distinguishing between mental rotation and perspective-taking processes (Wraga, Creem, Profitt, & Proffitt, 2000). The former includes those abilities related to manipulating images of spatial patterns internally and comprehending imaginary movements in 3D spaces (mental rotation processes), whereas the later to measuring abilities associated with spatial relations in which the body orientation of the observer is an essential part of the problem, for example, imagining how a stimulus array will appear from another perspective (perspective-taking processes). However, researchers have also noted a third strategy set in which individuals rely on the use of analytic processes to solve tasks based on non-spatial information (Linn & Petersen, 1985). In fact, studies in the early 1960s have already mentioned the possibility that respondents could use analytic processes to rotate figures when mental rotation becomes a more complex task than an analytic approach (French, Ekstrom, & Price, 1969).

Indeed, the distinction between spatial or 'holistic' strategies, which involve analytic reasoning rather than mental manipulation of objects, is very common in the spatial ability literature (Hegarty, 2010). Black (2005) reports a variety of domain-specific analytic strategies used by subjects in a laboratory-based task to solve spatial ability problems with little to no use of spatial information, concluding that once participants develop an analytical rule as they try to solve a problem, subjects switch to this rule-based strategy and no longer report the use of imagistic strategies to visualize a solution. Similarly, Stieff and Raje (2008) found that the majority of problem-solving approaches in experts solving chemistry problems were characterized by analytic strategies, whereas Linn and Petersen (1985) reported that in solving cross-section problems, those individuals who were able to develop a repertoire of strategies were also those who performed well on the problems.

Further, research by Keehner, Hegarty, Cohen, Khooshabeh, and Montello (2008) found that most subjects decompose tasks to solve cross-section problems: first they determine the outside shape of the cross-section, then they try to see how many ducts there should be in the drawn cross-section, afterward they determine the shape of the ducts and finally they infer where the ducts should be. Moreover, Hegarty (2010) argues that the best spatial thinkers are those who augment visualization with analytic strategies in order to visualize only the information that they need to transform and solve a problem. Overall, these studies suggest that both mental rotation and perceptive-taking processes are augmented by more analytic forms of thinking such as task decomposition and rule-based reasoning.

1.2. *Visualization Through a Problem-Solving Lens*

Problem-solving skills are commonly considered one of the most relevant professional cognitive activities and highly valued in contemporary learning theories (Sweller, 1988). The implementation and discovery of strategies for problem solving has been comprehensively examined in the education literature. Siegler (2006) explains that adoption of new strategies is slow since prior strategies persist even when the newly discovered strategy has clear advantages. Research has shown that individuals do not switch abruptly from using ineffective strategies to effective expert-like strategies, but rather employ more general strategies first (e.g. trial and error, means-ends analysis) for extended periods and then, as they slowly practice and increase their knowledge in the area, develop more heuristics and expert analytic strategies (Stieff & Raje, 2008).

Problem-solving can be loosely defined as a goal-oriented sequence of cognitive operations. The extant literature has distinguished two main groups of strategies in problem-solving: search-based mechanisms and schema-driven processes (Gick, 1986). Search-based mechanisms are used by novices, and the most general strategy within this group is the means-ends analysis, which is related to the reduction of the difference between the current state and the goal of the problem. Newell and Simon (1972) explain that problem solvers first understand the goal of the problem and then look for problem-solving operators. If the solution is successful, the task is over. If it fails, the solver backtracks to an earlier stage and attempts to redefine the problem or use another method to solve it.

Through schema-driven processes, on the other hand, experts are able to work forward immediately by choosing appropriate steps that lead them to their goal as they recognize the problem from previous experiences and already know the next moves to solve it. This schema-driven process is activated heuristically as a data-driven response to some related cue in the problem (Chi, Feltovich, & Glaser, 1981). Once the schema is activated, the knowledge contained in the schema provides the general steps and procedures to follow in order to solve the problem. The knowledge of experts thereby, should be considered as 'coherent chunks of information organized around underlying principles in the domain' (Cook, 2006, p. 1078), which is used to perceive underlying patterns and principles in problem situations. In contrast, novices virtually 'see' a problem differently since they do not possess appropriate schemas, and consequently cannot recognize problem configurations. Thus, they have to use more general problem-solving strategies such as means-ends analysis or processes that are based on hierarchical task decomposition (Gick, 1986).

1.3. *Visualization in Science Education: How can this be used for learning?*

Research on learning with visualizations has demonstrated that once learners interact with appropriate representations their academic performance is enhanced (Ainsworth, 2006). Two main theories in the problem-solving literature that bases instructional design principles on learners' cognitive structures have been used to explain the learning attained by students through visualizations (Cook, 2006): (1) the distributed cognition framework and (2) theoretical models that view visual and analytic reasoning as complements. The basic premise of the distributed cognition framework is that learning processes are hindered when the instructional materials overcome the cognitive resources. This framework holds that there is a cognitive architecture consisting of a limited working memory that interacts with an unlimited long-term memory (Sweller, 2004). Thus, the overload on working memory could be decreased by either augmenting its capacity or reducing its cognitive load. In fact, the external representations that subjects use for problem-solving is not just a marginal assistance to cognition, but rather, they are strongly related to the internal cognitive processes. By using visualization techniques, a distributed representational space is created in the subjects' minds, which could help them in turn, to solve a problem using these additional resources.

Regarding theoretical models that view visual and analytic reasoning as complements, the literature in problem-solving indicates that in order to construct a richer understanding of the processes that take place during problem-solving that involves visual representation use, learners need to integrate both visual and analytic thinking (Presmeg, 1992; Stylianou, 2002; Zazkis, Dubinsky, & Dautermann, 1996). The Visualizer/Analyzer (V/A) model, for example, holds that visualization and analysis, although distinct forms of thinking, inform one another and work together in the process of problem-solving (Stylianou, 2002). It is expressed in terms of discrete levels of visual and analytic thinking, as an approximation of the continuous thinking of the learners. The analysis described by the model begins with an act of visualization, V1. The next step is an act of analysis, A1, which consists of some kind of reasoning about the objects constructed in step V1. This is followed by a subsequent act of visualization, V2, in which the subject returns to the same 'picture' used in V1, but reasoning is enriched as a result of A1. That leads to a second act of analysis, A2, which is followed by a third step of visualization, V3, and so on. The iteration ends as the problem solver comes to a better understanding of the problem he/she is solving.

An extension of prior studies on problem-solving in the visualization area which analyzes how individuals learn from similar sets of problems and begin to develop their own problem-solving strategies is an appropriate next step to take. Our work begins to take this next step. As research has shown, cross-section problems provide an excellent opportunity to study the role of alternative strategy choice in problem-solving (Kali & Orion, 1996; Stieff & Raje, 2008).

1.4. *This Study*

We present a qualitative study designed to understand the learning process and strategies employed by individuals to visualize internal structures of 3D visualizations. The study aims to analyze the strategies developed by participants to overcome the complexity of diagram properties that prevent their comprehension. Based on previous findings that both mental rotation and perceptive-taking processes are augmented by more analytic forms of thinking and that, with practice, individuals develop and implement more heuristics and expert analytic strategies, we aim to answer two research questions:

> RQ1: What types of strategies do individuals develop to infer cross-sections of 3D objects?
> RQ2: Could individuals learn from using similar sets of problems to more effectively develop analytic strategies to solve visualization tasks?

The answers to these research questions are very relevant to us since the third goal of this paper and our overall research is to develop intelligent software able to enhance the learning process and the acquisition of viable analytic problem-solving strategies. Several studies have shown that visualization software can enhance 3D geometric proficiency by improving a user's abilities to build spatial images (Hauptman, 2010). Shneiderman (1997) has suggested the potential benefits of using physically based or tangible interfaces to enhance spatial skills and improve a user's abilities to build spatial images. Our research is based on the idea that cognitive benefits result from manipulating physical materials, and that mental processing benefits from using concrete physical objects designed to support more natural learning. Marshall (2007) explains that because tangible interfaces often utilize concrete physical manipulation, they might support more effective or more natural learning, consequently 3D forms would be perceived more readily through tangible representations than through visual representation alone. Further, consistent with the Cognitive Theory of Multimedia Learning (Mayer & Moreno, 1998), the advantages that external representations play in supporting learning are: (1) they reduce the amount of cognitive effort required to solve equivalent problems (Larkin, McDermott, Simon, & Simon, 1980); (2) they help to assimilate cognitively the content presented, as users interact with the figures virtually (which serves as a copy of the real world) (Mayer & Moreno, 1998); and (3) they hasten the learning process, as the virtual figures generated by the software gives the user intuitive freedom of action to interact in an unlimited manner with the objects in the virtual environment (Winn & Jackson, 1999). Thus, answers to both research questions will suggest design solutions for a software package intended to foster the strategies developed by the participants in our study

2. Methodology

Eleven undergraduate students were videotaped as they attempted to solve 18 'slice' visualization problems. They were asked to visualize the internal structure of 3D visualization diagrams by drawing what the cut face of the visualization would look like if it were cut internally along the dotted lines shown on the figure. Each slice visualization problem was drawn on the top part of an individual sheet of paper. The problems were ordered in increasing difficulty as determined by a panel of experts in this type of visualization. Table 1 shows all the slice visualization problems we used where the first exercise is the easiest one and the 18th is the most difficult one.

Subjects were asked to draw the cut plane of the 3D visualizations on the bottom part of the sheet of paper, while thinking aloud and expressing the difficulties they faced. This process, which is known in the literature as a think aloud protocol, has been used to examine the cognitive processes in different disciplines, and it was chosen because such verbalizations present an opportunity to make students' reasoning more coherent and reflective (Ericsson & Simon, 1998). In our study, the interviewer gently reminded participants to 'keep talking' while solving the problems. When subjects finished drawing the slice, the interviewer asked participants to review the steps followed in order to get more information about the processes behind their actions. Then the solution to the task was given and participants were allowed to compare it with their answers. Subjects were asked to stop after an hour even if they did not finish the 18 problems. Ten subjects did not complete the entire set of problems.

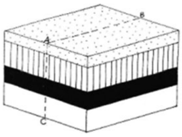
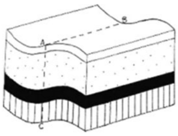
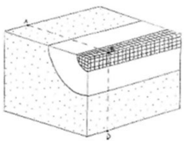
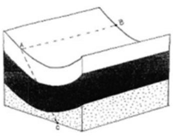
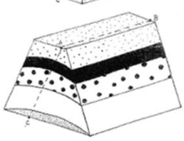
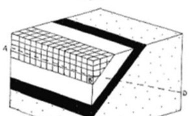
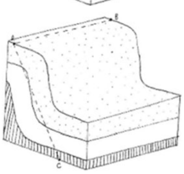
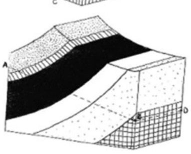
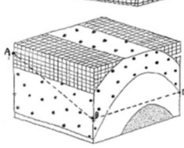
2.1. *Participants*

Participants were students from Communication departments and Engineering departments at our university. We focused on recruiting half of our participants from non-science disciplines since such individuals are likely to have lower spatial abilities. Among the 11 participants, 6 were male and 5 were female. Their ages ranged from 18 to 26.

2.2. *Generation of 3D Visualization Diagrams*

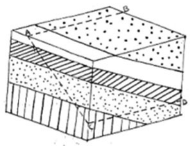
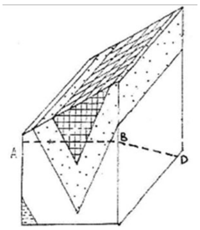
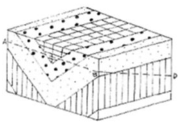
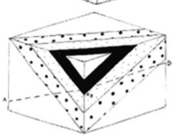
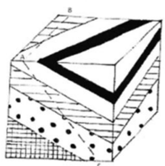
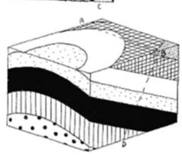
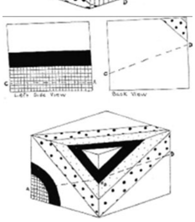
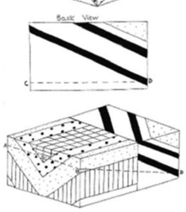
The visualizations used in this study were black and white 2D representations of 3D diagrams with different patterns used to represent internal layers. The 3D visualizations had virtual slices drawn through them that were represented by the letters A, B and C connected by a dotted line, as is shown in Figure 2. The patterns can be observed in the figures presented in Table 1. The diagrams contained multiple properties that represented features of visualizations that typical cross-sections contain (Kali & Orion, 1996). To generate the diagrams, a set of representative 3D visualizations were collected from introductory Geology and Earth Science textbooks. The slice visualization problems were ordered from easiest to hardest for presentation to participants by asking seven experts and two novices to solve 32 visualization problems and rank-order these problems by difficulty. Group consensus was then used to select 18 ordered problems in which all difficulty judging participants agreed on their order.

Table 1. Descriptive data for slice visualization problems

ID	Diagram	Corrects	Perspective taking	Spatial transformation	Analytic strategies	Developing strategies
1		11	7	0	1	0
2		11	8	2	1	0
3		10	12	2	2	0
4		11	8	3	3	0
5		10	15	4	5	0
6		8	25	5	12	0
7		6	16	5	7	0
8		7	23	7	14	0
9		6	22	6	21	2

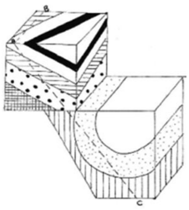
(*Continued*)

Table 1. Continued

ID	Diagram	Corrects	Perspective taking	Spatial transformation	Analytic strategies	Developing strategies
10		5	21	6	23	2
11		4	20	6	22	2
12		4	15	5	19	2
13		3	11	4	14	2
14		2	11	4	13	1
15		2	10	4	12	1
16		1	8	2	10	1
17		1	7	2	9	1

(*Continued*)

Table 1. Continued

ID	Diagram	Corrects	Perspective taking	Spatial transformation	Analytic strategies	Developing strategies
18		1	7	2	9	1

Note: The table shows how many participants drew accurately each one of the 18 diagrams and the number of strategies employed for this purpose.

2.3. *Operationalization of Key Terminologies*

- 3D visualization problems: 18 3D visualization diagrams were given to participants.
- Planes of presentation: A viewer's automatically assumed *x*, *y* and *z* coordinate system (Heo & Hirtle, 2001).
- Cut: The plane that slices 3D visualizations into two pieces. It is often referred to as a cross-section in different fields (Cohen & Hegarty, 2007; Kali & Orion, 1996).
- Action: Any movement, gesture, sketching or verbalization generated by the participant while solving the 3D visualization problems.

2.4. *Data Analysis*

In order to identify inductively the problem-solving strategies developed by participants, verbal protocols as well as subjects' gestures and drawings were analyzed. For this purpose, we used Transana (http://www.transana.org/), a qualitative analysis

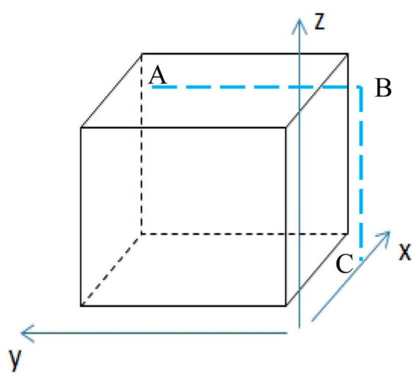


Figure 2. The *x* and *y* axes form one plane. The *x* and *z* axes another with the *y* and *z* axes the third plane. Because the figure is presented as a block, the viewer automatically assumes that the shown *x*, *y* and *z* axes are the coordinate system in use. The dotted line through the block shows the cross-section or cutting plane

software for video and audio data. The transcription of each interview was analyzed using established techniques in previous verbal protocol studies (Ericsson & Simon, 1998). Each problem-solving exercise was regarded as an independent unit, and each participant's utterances and behaviors trying to solve the problem were analyzed to identify the actions used. We analyzed the data in four steps. First, we transcribed each of the video recordings, and as we analyzed the transcription, we identified patterns employed by subjects to solve the tasks in order to have a narrative depiction of the actions. Thus, this first phase of data analysis was designed to identify actions that ultimately would be leading participants to develop a strategy. We began the analysis using the process of theoretical coding (Glaser, 1978) by examining the videos and the transcripts actions that appeared to lead participants to problem solutions, and applied codes to these instances. In total, 24 distinct codes were produced.

The next step of analysis was axial coding. This involved reassembling the coded data in new ways by grouping codes that were conceptually similar (Charmaz, 2006). The third data analysis phase was designed to confirm the codes utilized to categorize the actions. For this purpose, each video recording was viewed more than five times by three researchers until all agree on the actions employed by participants to solve the problems. As we observed the video recordings multiple times, a table was developed based on the agreement of the researchers. A portion of this table is presented in Table 2, where it can be seen as a description of the actions developed by participants, the physical gestures used to fulfill the actions and some excerpts of the transcription that illustrate how the subjects verbalized their efforts to solve the problems by using these actions.

Finally, to corroborate the coding, we followed the approach taken by Stieff and Raje (2008) in order to identify strategies. First, the frequency and order of the actions were analyzed to understand which specific actions led subjects to develop particular strategies. We compared the tables of actions employed by each participant in order to identify sequences of actions before participants began to implement a strategy. In this way, we also could see whether subjects learned from earlier problems and used this learning to develop strategies that they then applied to their next visualization task. Once we finished the comparison of the actions employed by participants, we could determine whether students were employing similar strategies as will be shown below. It is relevant for us to note that after participant number eight, we did not find any action or new strategy employed by the three last individuals. Thus, after 11 videotapings, we decided to not videotape more participants. We may have missed more potential strategies, but felt that the accumulations of knowledge that we had already obtained was sufficient to begin experiments to further verify our results and also being the development of a sequence of visualization slice problems that would aid in the learning of these analytic strategies.

3. Results

Only one of the subjects completed the 18 exercises accurately in 1 hour. Two participants completed 14 problems, two subjects solved 10, two participants solved eight

Table 2. Example of a table developed while analyzing data in one of the subjects until problem eight

#F	Actions identified in participant	Gestures made by the participant	Excerpt of the transcription in each action
1	Compares the cut/slice with the front face of shape	Indicates the front	If I look at here, I should know where the cut starts
1	Compares angles to estimate the portion of the pattern	Moves from the cut to the front face	There is perspective here, it should be tall here
2	Compares the cut/slice with the front face of shape	Uses fingers to measure	Through this way I know the proportions are ok
3	Rotates head to visualize the slice and patterns of diagram	Physically rotates/ moves the paper	So we need to rotate it
3	Rotates figure to solve problem (put the cut vertical)	Rotates paper	Like the other images I will have to rotate …
4	Comparing the cut/slice with the previous ones	Indicating the cut/ slice	This looks like a cube and it should be a rectangle
4	Draws lines checking figure with proportion in diagram	Marks the diagram to measure	Through this way I can calculate better the size
4	Identifies the easier point of view to visualize the image	Looks the diagram and moves it	Before the way the angle was cut was from behind …
5	Rotates in his mind the cut from horizontal to vertical	Rotates paper	I could rotate it but now I feel ok visualizing it from the top
5	Contrasts the position of the lines with the rest of the figure	Draws lines parallel to the figure	This layer has the lines, I'm getting a little confuse … .
5	Draws lines to visualize the rotation of the figure	Draws outside the figure	Through this I can understand the cut
6	Compares the faces of the figure in order to gain information	Looks the overall shape of the diagram	It looks pretty much like a cub and I can gain information …
6	Identifies what the cut would include based on the front face	Shows the parts he is not including	I'm not concerned about this because I assume it's under the cut
6	The use of external elements/ experiences to visualize the cut	Draws a figure outside the diagram	I'm imagining a cake

Note: The first column shows the action employed by the subject, the second one the gestures and the third column shows excerpt of the transcription in each action.

and three participants solved up until problem number 6. Problem number 4 was the last one that all the participants solved it accurately. Table 1 shows the different problems, how many participants accurately drew accurately each one of them and the number of strategies employed by each participant for this purpose.

Answers were considered correct if the general pattern of the layers was accurate. Inaccuracies in depicting the thickness of layers, or their precise location in the cross-section, were also ignored. Figures 3 and 4 show accurate and inaccurate depictions of the cut face draw by participants. Our analysis found subjects employing a variety of actions and strategies to visualize the internal structure of the 3D diagrams with diverse degrees of effectiveness. The results indicated actions related to perspective-taking, spatial transformation and analytical processes.

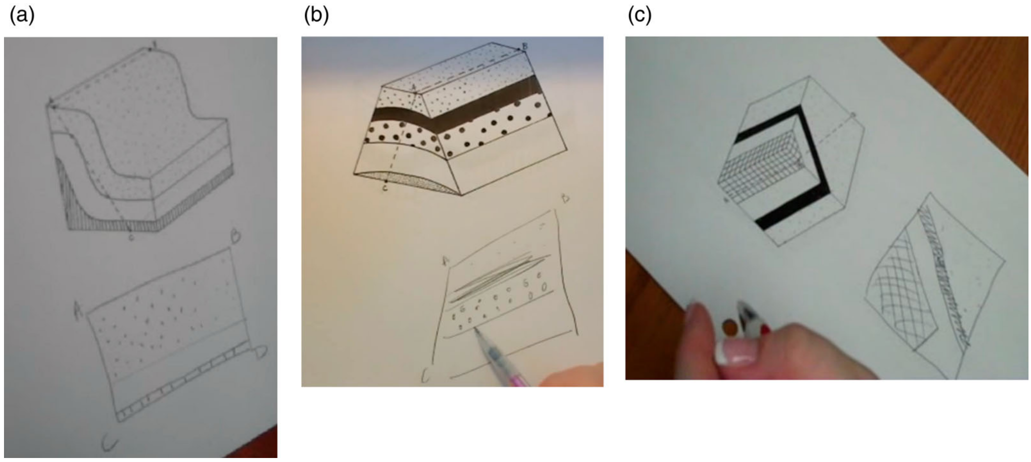


Figure 3. It shows the most frequent errors made by subjects. (a) Illustrates how the subject did not consider one of the layers. A similar error can be observed in (b). The drawing in (c) shows an error with the perspective adopted by the subject, who could not visualize the angle of the cut face and only copied one of the faces

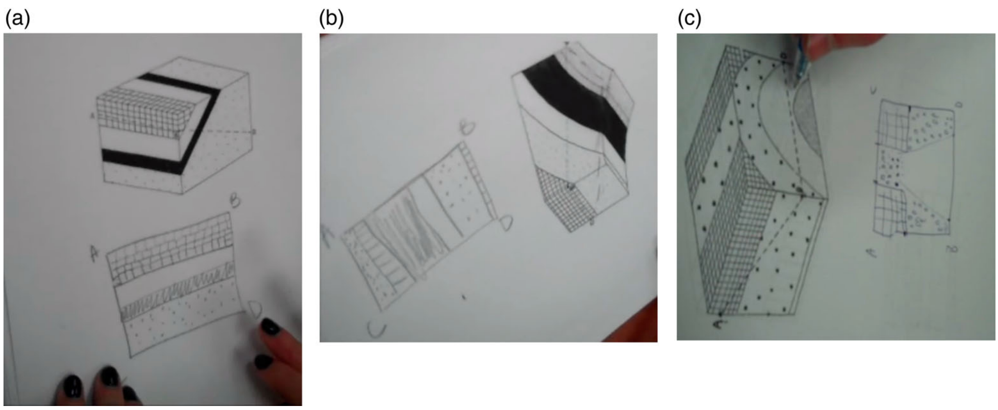


Figure 4. These examples show accurate drawings of the problems, although the thickness of layers or their precise location in the cross-section were not 100% correct: (a) shows the first layer wider to what it should be, in (b) the last layer should be thinner, and in (c) the form of the layer with cubes should be more in diagonal

Regarding the analysis, axial coding resulted in a reclassification into eight main codes indicating actions related to three categories. The most common one, *perspective-taking strategies*, included three actions: head rotation (the rotation of the participant's head in one direction or another), sheet rotation (the rotation of the sheet of paper containing the problem) and letter labeling (participants labeled the letters in the problem's solutions in order to orient the solution regarding the problems. The second category, *spatial transformation strategies*, only included the mental rotation of the problem (this was learned through the verbal protocol). The third, *analytic* *strategies*, considered four actions: blocking information to see a portion of the figure without interference from other parts, drawing what would be external extensions of the patterns shown on the outside of a figure, measuring and contrasting the lines of the figure with the drawing to make it proportional (by comparing the relative sizes and relationships of visible features and using these to determine the sizes and relationships of features in the slice) and actions related to the decomposition of the problem, such as dividing the exercise into parts as will be shown below.

3.1. *Perspective-Taking Strategies*

We define a perspective-taking strategy as one in which the participant attempts to change his or her point of view of the 3D object in an attempt to make the problem easier to visualize. Perspective-taking-oriented actions were the most simple and frequent strategies found in the study, as can be observed in Table 1. We identified three categories of actions related to this strategy. The most common action was changing the position of the head in relation to the problem. All the participants, at some moment during the study, rotated their heads in order to virtually arrange their position to be one parallel to the front face of the diagram. For example, they either tipped their heads sideways to the left or right in order to be looking at the 3D figure from the left front side or right front side, respectively. The second action was the rotation of the problem sheets orienting them to different positions in order to look for a better angle that allowed them to see parts of the problem in a new x–z plane. This was observed in 10 of the participants.

Participants were aware of choosing a different perspective to visualize the problem. Subject 3, for example, once she finished problem 9 and was asked about the actions she used it in order to solve the problem, stated that she used a top-down approach to visualize it. 'I felt much more comfortable seeing the slice from here [showing the top face] than from the bottom, since it is easier for me to take this part off [showing the part below the dotted lines]'. Similarly, eight of the participants mentioned, at least in one of the exercises, that they had changed their perspective regarding the representation of the object. Interestingly, seven of the eight participants who used a self-referential position to visualize the problems employed this action when they solved problem number 6, the first task in which the cutting plane was both horizontal and at an angle (this is shown in Table 1). In the prior problems, participants could imagine themselves viewing the problem solution by either positioning in front of the x–z plane of the figure or in front of the x–y plane. With problem 6, they needed to imagine themselves looking down on the figure or up at it. Problem 6 records the highest number of actions (25) related to this strategy, as shown in Table 1, demonstrating that participants had to employ these types of actions in order to accurately solve the problem. Eight of the 11 participants solved this problem. It was in this exercise that subjects started to mention that they were taking a bottom-up or top-down view, instead of using a frontal perspective. Participants who showed a higher flexibility in shifting their referential points and looked at the figures from different perspectives performed better than those who did not indicate this skill: Seven of the individuals who applied this strategy in problem 6 accurately solved problem 8 (which required a similar approach).

The third perspective-taking action identified was used to determine the relative position of the spatially arranged items in the diagram. Ten of the subjects labeled each corner of their slice with the letters ABCD, imitating the original cut shown in the problem. This lettering served as reference points and was used to orient the subjects. Participants gave a similar reason to explain their actions: 'Putting the letters helps me to know where I am when I compare my drawing with the figure and how I need to continue' (Subject 3); 'It is easier to know in which part of the exercise I am and the perspective from where I'm solving the problem' (Subject 5).

3.2. *Spatial Transformation Strategies*

Spatial transformation strategies are those which focus on manipulating images of spatial patterns internally and comprehending imaginary movements in 3D spaces. The only action identified was a 90 degrees rotation of the problem using the letters on the cut plane of the diagram as a point of reference. In fact, as Table 1 shows, this was the strategy least used by participants. The other spatial transformation actions were mixed with analytical processes as will be explained in the next section. In problem 6, for example, as was mentioned before, two subjects indicated explicitly at the end of the exercise that they rotated the block mentally. Subject 6, for example, explained: 'I think it's much more difficult for me to see horizontal cuts because we don't cut cakes horizontally, we do this vertically, and I think I am more used to that'. The action used by participants to transform the figure spatially was to draw a square/rectangle representing the cut plane and copy the letters of the cross-section as points of reference on to this plane. Then, based on these points of reference, rotate the shape of their drawings (from horizontal to vertical) in their mind. This strategy was recognized in eight participants. Subject 1, for example, said: 'I have a real need for rotation, I need the rotation if I have B here … So I'm going to stick to the same A, B, D, C case here, that will be easier for me'.

However, it is important to note that after problem 9, none of the participants were able to mentally manipulate the cutting plane without external visual aids (e.g. by drawing an additional figure representing a portion of the original figure) or by using a learned analytical process (task decomposition). In fact, those who tried to solely visualize the problems in their minds could not solve them without this additional external help. Participant 1 tried first to rotate the figure mentally, but soon realized he could not visualize the problem internally: 'This cross section has a layer that should come into view on the other side [trying to picture the other side] … but I don't know … I guess it is difficult'. Subjects who relied only on internal processes were so concentrated on visualizing the cross-sections that they could not even correctly verbalize their thoughts. For example, participant 5 said, 'It is cutting it this way [pointing at the cut with the pen], sort of [pausing]. Hmm … not actually sure [pausing] … Uh … [pointing at the cut with the pen again] I can't really tell it's cutting at a diagonal'.

3.3. *Analytic Strategies*

Analytic strategies involve reasoning rather than mental manipulation of objects in which the problem solver uses a general rule to create a solution. It is important to note that most of these actions were employed as the difficulty of the problems increases. In fact, Table 1 shows that during the first problems none of the participants applied them. We identified three main categories of analytic strategies. For the first category, we grouped two actions intended to reduce the memory load when visualizing internal patterns (layers) crossed by the cutting plane: (1) The blocking of irrelevant information by participants—most of the subjects used their hands to physically enclose the cutting plane and hide the rest of the problem, as Figure 5(b) and 5(c) show—and (2) The creation of extended lines to mark out the area enclosed by the cross-section, as is shown in Figure 5(a). These two actions were employed by 10 of the subjects, and some of the participants even developed a sequence of actions to enclose the cross-section, as Participant 3 noted:

> I wanted to do the same thing, to draw this face, and mentally block the rest [covering the diagram with her hands], but then I tried to do a new thing, to draw the invisible line in order to make it easier to get the slice [she draws lines over the cross section uncovering the lines of the plane].

When participants were asked why they employed these actions, they indicated that they felt confused by the additional sections of the block diagram and wanted to avoid too much information by focusing only on the cross-section.

The second category includes those actions directed at measuring the initial and final layers that were included in the cross-section. This was done to assess possible changes in the patterns of the layers. Seven of the subjects marked reference points in the slices they were creating before drawing them in their answer in order to estimate, based on the marks, how the route or trajectory of the layers would change.

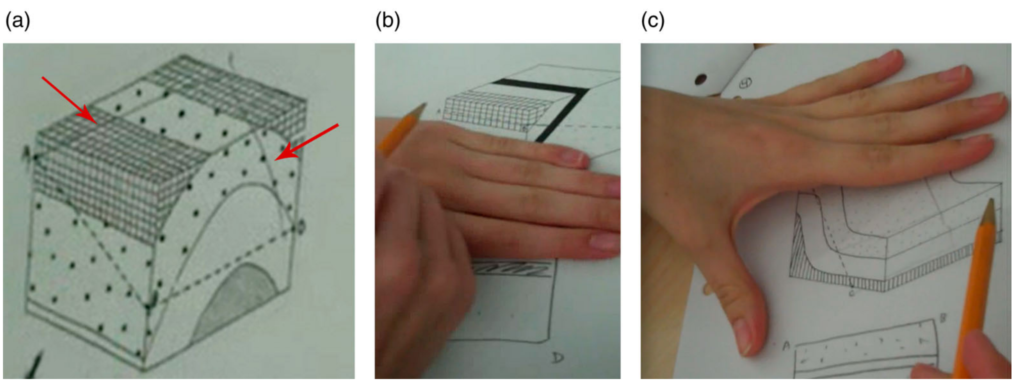


Figure 5. The arrows in (a) show the 'invisible lines' extended by the subject in order to visualize the cross-section, whereas (b) and (c) show a subject blocking the information not used in the diagram to enhance her vision on the problem and infer the cross-section

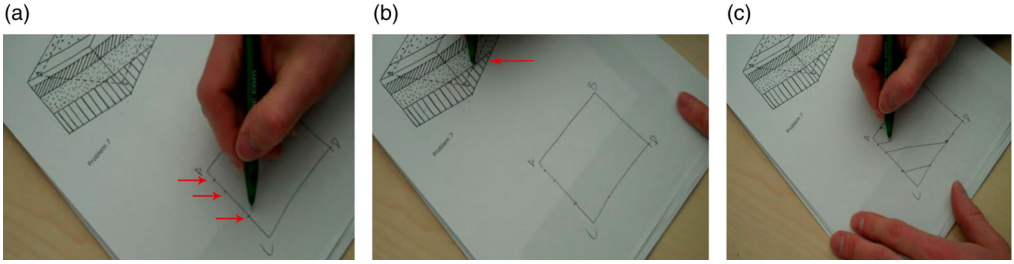


Figure 6. The arrows in (a) note the first 'reference marks' that represent where the layers should start. (b) Shows the subject assessing in the block diagram where the layers should end, and (c) shows when the participant connects the marks forming the layers

Their procedures were very similar: first they compared their drawing slice with the block diagram, marking in each side of their slices where the layers should start, as can be seen in Figure 6(a). Five of the individuals who applied this strategy started from the corners of their drawing slices and then continued toward the center of their figures. Then they checked the different faces of the blocks in order to confirm the proportions, as Figure 6(b) shows, identifying what they believed would be the end of the layers. Finally, they connected the points marked previously, showing the final routes followed by the layers, as is illustrated by Figure 6(c).

The third category identified consisted of a series of actions in which participants decomposed the problem into smaller units in order to visualize the cross-section through progressive steps. After problem 11, all the participants who accurately drew the cross-sections used this approach, especially in the most complicated problems which contained embedded figures, as is shown in Table 1. This strategy was chosen in response to previous failures, since all the participants who employed the decomposition technique first tried to visualize the layers encompassed by the cross-section directly and simultaneously, but then, when they realized that they were not succeeding with this approach, they decided to modify it. As subject 7 noted: 'I couldn't do everything at once, I couldn't see everything so I had to breake it into small pieces in order to go step by step, approaching each part differently'. Participants typically tried to first determine the basic figures or less complicated areas of the diagram that were part of the cross-section; then they looked at how the layers might change based on the angle of the cutting plane; next they would determine the cross-section shape of the more complicated figures; and finally they inferred what the overall cross-section might look like. This set of analytic actions matches what Hegarty (1992) calls task decomposition. Table 1 shows that participants who did not use these actions could not solve the advanced problems.

3.4. *Developing Problem-Solving Strategies*

This section presents the analytic strategy developed by two participants that was used to understand the configurations of the layers. Both subjects arrived at this technique

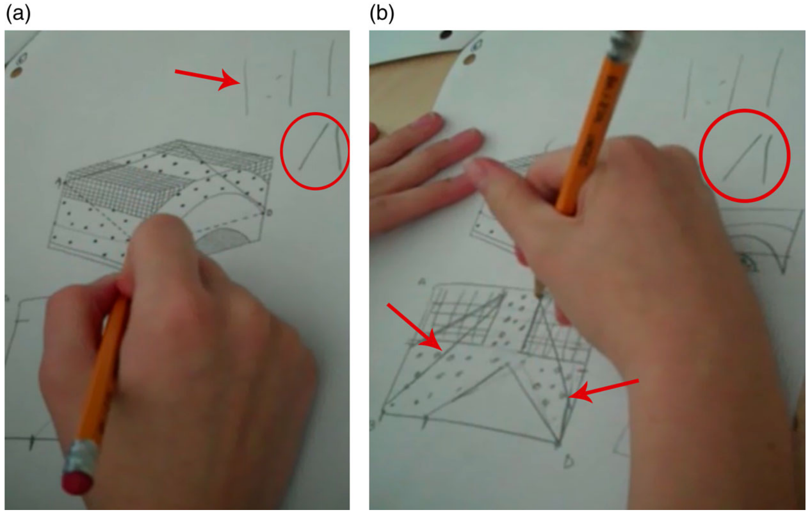


Figure 7. It shows the sequence of actions in problem 7 that lead a subject to develop the strategy. The red arrow in (a) shows the drawing made to visualize the layers of the top face of the block diagram, whereas the red circle indicates the figure through which she could picture how the layers would be seen when penetrating the block diagram. (b) Notes the application of this logic in her drawing slice, and how she changed the squares (layers without penetrating the figure), for an angle

by drawing an extra figure that supported their analytic process. The strategy was developed after several attempts to find a perspective from which they could visualize the cross-section shown in problem 9, the first problem that required integrating three faces of the block diagram to visualize the cross-section. In both cases, participants drew a new figure using only the top face of the diagram as a reference (as the red circle and arrow show in Figure 7(a)). Then they used this figure to imagine how the layers of the top face would penetrate the block diagram when the slice was at an angle.

Figure 7(b) shows the first pattern of layers drawn by one of these subjects, before she began to implement her integration strategy. This was based on the visualization of the external patterns exposed on the top face of the block diagram. The 'squares' pictured in each corner of her diagram indicate the difficulty the participant had in envisioning how the internal layers of the structure would change with the angle of the cutting plane. But then, when she was explaining the steps followed to solve the problem, she remembered the external figure she had drawn a few minutes before when she was using the top face as a reference. Immediately, she decided to change the 'squares' that she had drawn to be 'triangles', reflecting what she had learned from her external figure. The red arrows illustrate this in Figure 8(b).

When participant 7 started the next exercise that also required integrating the three faces of the block diagram to draw the cross-section (problem 9), she did not immediately apply the strategy just learned. First she tried to solve the exercise employing the same analytic techniques she had been using before the development of this new

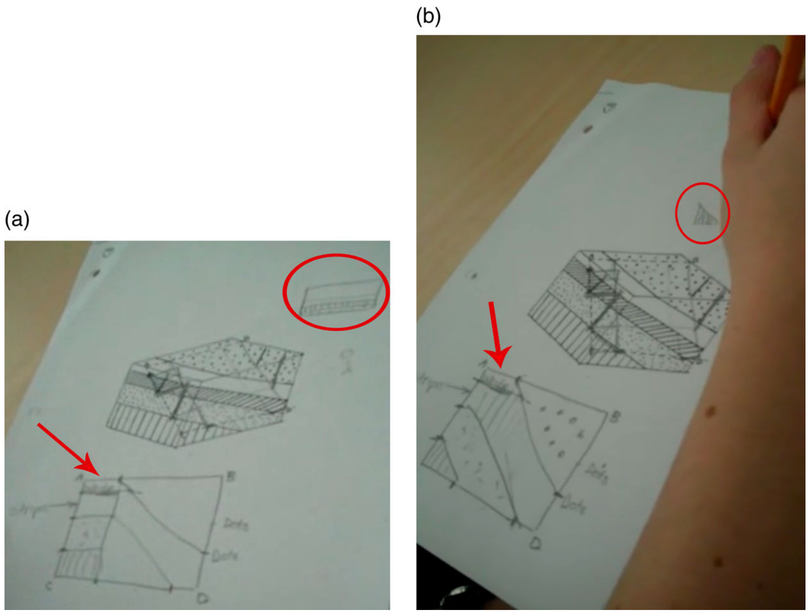


Figure 8. The red arrow in (a) indicates how the subject visualized first the internal structure of the block diagram without applying the strategy, whereas the red circle shows the drawing made by the subject reflecting the top face of the block diagram. The red circle in (b) indicates the figure through which she could picture how the layers would be seen when they penetrate the block diagram, and the arrow shows how she changed the squares (layers without penetrating the figure), for an angle

strategy, for example, marking points of reference to calculate where the layers should start and end in the drawing slice. As can be seen in Figure 8(a), the subject first drew squares in the block diagram between the layers and the angles, in order to know how wide each layer would be. Then she drew similar squares in the left side of her drawing, as bench marks, to visualize from which points the layers should be draw. This is indicated by the red arrows in Figure 8(a). However, after 10 minutes, she encountered the same difficulty in envisioning how the internal layers of the structure would change in relation to the angle of the cutting plane.

At this point, the subject focused again on the top face of the figure as she had in the previous problem, but this time she tried to visualize how the internal layers of the block diagram would be seen if they would be leveraged or raised to the top face. In other words, she tried to see the top face of the block with the internal layers projected on this face. However, since it was difficult for the participant to picture in her mind how the internal layers would be seen projected on the top face of the diagram, she drew an extra figure to visualize this projection. This is reflected in the red circle in Figure 9(a). Once the subject could visualize how the top face of the block would be seen with the projected layers, she repeated the procedure she had developed in problem 7. She drew a new figure using the top face of the diagram and then started to imagine the 'top face' layers penetrating into the figure. And then, after

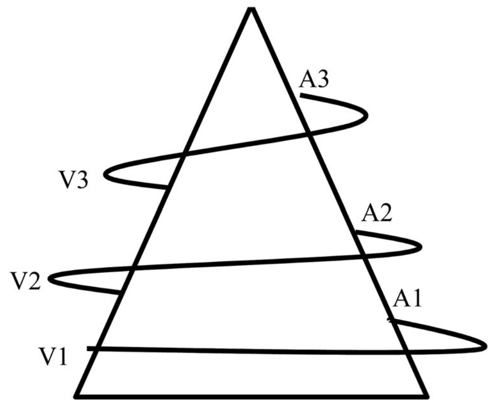


Figure 9. An adapted Visualyzer/Analyzer model. In a subsequent act of visualization, V2, participants used the same 'picture' used in V1, but as a result of the analysis in A1, in which they calculated the 'bench marks' to see where the layers would start, it has changed. That is, in V2 compared these marks with the block diagram for accuracy, and A2 lead participants to the construction of a new picture to contrast how the layers would be affected by the angles. And then, based on these measurements, they went back to check the different faces of the blocks in order to confirm the proportions (V3), by identifying what they believed would be the end of the layers (A3), to finally connect the points marked previously. In any case, the result is an external representation in which the individual achieves some richer understanding of the original situation

she could picture the patterns of layers on the top face, she tried to visualize the internal structure:

> I started doing top-down, like sinking into the image, I started to imagine peeling this face, and then slowly go down into it, and then I realized that it's going to be into it, and it's going to be replaced by the other pattern.

Once she could picture how the layers would change with the angle, she drew a new figure reflecting this new pattern, as can be seen in Figure 9(b), and with this approach she correctly solved the problem and repaired the original slice she had drawn 'It would be like a little triangle, and then slowly it's going to be bigger, and bigger, and so another triangle came'. Interestingly, after this problem the subject definitely learned this strategy and incorporated it in her pool of techniques, since she used it immediately in the next two problems, skipping the analytic techniques employed previously. In problems 7 and 9, it took 12 and 10 minutes, respectively, to solve the problems, whereas in problems 10 and 11, when she implemented the strategy immediately, it only took her 2 and 3 minutes, respectively.

4. Discussion

This paper focuses on identifying which viable strategies students use to infer and interpret cross-sections of 3D objects. We have carried out the study with the intent of designing visualization software problems able to support the learning processes exhibited by our participants. Our findings suggest that although spatial transformation and perspective-taking techniques were immediately used by our participants to visualize cross-section problems, both such processes were significantly augmented by analytic thinking. Subjects who employed analytic techniques were able to visualize more accurately the block diagram cutting planes than those who did not follow such strategies. Our research also showed that visualization of cross-sections is an effortful process: participants could not mentally imagine the configurations of the layers when the angle of the cutting plane was not perpendicular to such layers. They could only do so with external help (e.g. drawing an extra figure) or through analytic strategies. Finally, our analysis also indicates that participants employ general analytic strategies (trial and error) for extended periods and then slowly adapt more focused strategies with practice until they arrive at a set of workable expert analytic strategies.

4.1. *Theoretical Implications*

Our findings have implications for two main theoretical approaches in the visualization and problem-solving literature: (1) the distributed cognition framework and (2) theoretical models that view visual and analytic reasoning as complements. Regarding the distributed cognition approach, the study indicates that only participants who used an external element (a visual–spatial representation) to support their internal computation can mentally manipulate the structure of the block diagrams and the mapping of their layers on to the cross-section plane. These findings are consistent with distributed cognition theories, which hold that people tend to offload internal visualization processes onto the manipulation of external representations: all the participants during the study decided at some point to off-load their cognitive efforts to perceptual-motor actions such as using an external visualization, or simply rotating the paper (instead of rotating the block diagrams mentally), showing a preference for less cognitive effort as the minimum memory model predicts. Our results are also aligned with previous findings in the problem-solving literature, which suggests that as individuals learn new strategies, individuals apply schemas and draw inferences from new information in problem-solving situations (Larkin et al., 1980).

Regarding theoretical models that view visual and analytic reasoning as complements, our findings corroborated that visualization and analytic thinking can be considered two interacting modes that are complemented by each other in problem-solving tasks. Specifically, our results support the V/A model, which considers visual and analytic reasoning as counterparts (Presmeg, 1992). Interestingly, the intertwined pattern proposed by the V/A model was seen in most of the subjects who engaged in analytical strategies, specifically in those participants who used them to manipulate the patterns that appeared on the cross-section when the cutting plane was at an angle to the planes of reference. Figure 6 illustrates this process, in which participants start to draw regular layers visualized in the block diagram (V1); then they calculate the 'bench marks' to see where the layers start

(A1); consequently they compare these marks with the block diagram for accuracy (V2); to contrast how the layers would be affected by the angles (A2), and then, based on these measurements, they go back to check the different faces of the blocks in order to confirm the proportions (V3), in order to identify what they believe would be the end of the layers (A3), to finally connect the points marked previously. Figure 9 adapts the V/A model by describing the different steps employed by participants.

Lastly, the analyses of the strategies developed by participants were also consistent with findings in the use of strategic processing in academic performance. Research has shown that academic performance is highly related to students who monitor and regulate their cognitive processing appropriately during task performance (Chi et al., 1981). In our study, participants who reviewed the steps followed to solve the problems were able to develop and improve their performance, as was illustrated by the description of how participant 7 developed her strategy.

4.2. *Practical Implications*

Based on the cross-section visualization strategies employed by the participants, and supported by previous experiences that have demonstrated how visualization software can enhance 3D geometric proficiency by improving user's abilities to build spatial images (Hauptman, 2010), we suggest five main aspects to be considered in the design of the software aimed to facilitate learning effective internal processes in visual comprehension:

First, we found that subjects physically or mentally rotated their block diagrams to help in visualizing the cross-sections from their own perspective: participants preferred to arrange the 2D figure so that the X–Z plane was parallel to them instead turned at a 45 degree angle from this position. We also saw that the same block diagrams—especially those with a horizontal cutting plane—were visualized differently by subjects: with participants mentioning trying to visualize the cross-section by looking at the problem from the top of it (that is, in the X–Y plane); others attempted to visualize the diagrams from the bottom (still the X–Y plane). Overall, participants who showed more ability to change their perspective while solving the problems had consistently higher performance. In fact, Table 1 shows that after problem 9, only subjects who used one of these strategies could solve the remaining problems. From this analysis we can deduce that it may be useful for users to control their perspective when viewing 3D problems in order to more accurately visualize a cross-section. Consequently, the first solution for aiding the student in problem-solving suggests a computer application capable of rotating 3D figures in which the users have the opportunity to rotate the block diagrams but not by stepwise rotations, but rather by the selection of a viewing perspective.

Second, the analysis showed that it was helpful for subjects to put the letters (ABCD) as points of reference in drawing the cross-section, imitating the original positions of the cutting plane in the block diagram. By using the reference points, subjects were able to better comprehend the arrangement of the figure and understand their spatial relationship with the other elements in the block diagram. Also, by comparing their drawing with the original diagram, it was easier for them to move and rotate the drawing to fit the block diagram orientation. Consequently, the software application should support the use of references (e.g. cardinal points) to orient and guide users in determining the relative position of the spatially arranged items in the block diagram.

Third, our study indicated that participants often became confused trying to distinguish which elements to include in their cross-section drawing because the density and complexity of irrelevant features in the block diagram interfered with their selection. This 'information overload' effect was observed to cause subjects to employ strategies such as (1) blocking the non-useful areas in the diagram to help them in inferring the cross-section, and/or (2) drawing extensions to the cutting plane in order to clearly define the area included in the cross-section. Based on these responses, the software learning application should allow problem solvers to highlight those areas included in the cutting plane (e.g. marking them with a different color). It should also provide a design feature in which problem solvers can highlight different features and areas of the block diagram giving users the possibility to observe separately the different features that might be included in the cross-section. This design would also support the task decomposition strategy employed by many subjects.

Fourth, we found that participants who reviewed the steps followed were able to develop and improve their performance. This is consistent with findings in education research, which has demonstrated that students who relate their understanding to principles found in their texts and revisit their own understanding of problems learn and perform better. Thus, we recommend integrating a tracking function in the design of the software application that records the steps and approaches employed by users as they attempt to solve problems and gives users the capability to playback their prior steps. We envision implementing this as an undo and redo function. The design of this capability is also consistent with hypothesis-testing theories, which suggests that by providing problem solvers with feedback on their iterative process, they have more chances to learn viable solution strategies.

Finally, our analysis shows that for all subjects, it was difficult to form a mental image of a solution when the cutting plane was at an angle to the planes of reference. Only those participants who drew additional external figures and used analytical techniques were able to envision how the internal layers of a structure would change when projected at an angle on the cutting plane. The set of actions followed by subject 7 in our study suggest the following design for our 3D learning system. Subject 7 systematically approached the design by looking at how patterns on the top face of the block might be projected on the cutting plane, then how patterns on each of the side faces might be projected if the cutting plane were perpendicular, then changed by 10 degrees from the perpendicular, then 20 degrees and so on. To copy this approach, our learning system will involve giving multiple problems that are 'tweaked' slightly and presented in succession to the problem solver. The first problem would have a cutting plane that is perpendicular to the presentation planes. The follow-up problem would be the same but use a cutting plane that is set at 10 degrees from the perpendicular. The next problem would have a cutting plane set at 20 degrees and so on. This procedure would be employed with simple block diagrams and then proceed to more complex diagrams. Similarly, we will present problems in which the cutting plane is parallel to the *z*-axis and perpendicular to the *x–y* plane of the problems but also switch to cutting planes that are parallel the *y*-axis.

Overall, the learning software we are proposing will track the success of users in solving progressively more difficult 3D slice visualizations and adapt its problem presentation to progressively present those visualizations which the user is having the most difficulty with beginning with easier problems that slowly get more difficult and thus, suggest the analytic strategies to solve them.

5. Conclusion

The study investigated the range of strategies individuals develop to infer and interpret cross-sections of 3D objects with the goal of building training applications that integrate these strategies. Our results suggest that although spatial transformation and perspective-taking techniques are useful for visualizing cross-section problems, these visual processes are augmented by analytical thinking which evolve through practice into a set of progressively more expert strategies. We believe the conclusions obtained in the analysis may be useful to integrate into the design of intelligent software that facilitates internal processes such as visual learning and comprehension. Overall our investigation yielded five major findings for this purpose. First, the software should allow users to control the perspective through which they view 3D information in order to more accurately visualize the cross-section. Second, it should support the use of reference points to orient users in determining the relative position of the spatially arranged items in the block diagram. Third, it should provide a design feature in which the problem solver can highlight different features of the block diagram in order to enable users to observe individually the different features that might be included in a cross-section. Fourth, it should have a tracking function that records the steps and approaches employed by users as they attempt to solve problems and to allow users to playback the steps taken. And lastly, we believe that the software should incorporate an analytical technique to aid the user in envisioning for example how the internal layers of a structure change when either the cutting plane or the features change orientation.

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