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Activity

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Discovering Symmetry in Everyday Environments: A Creative Approach to Teaching Symmetry and Point Groups

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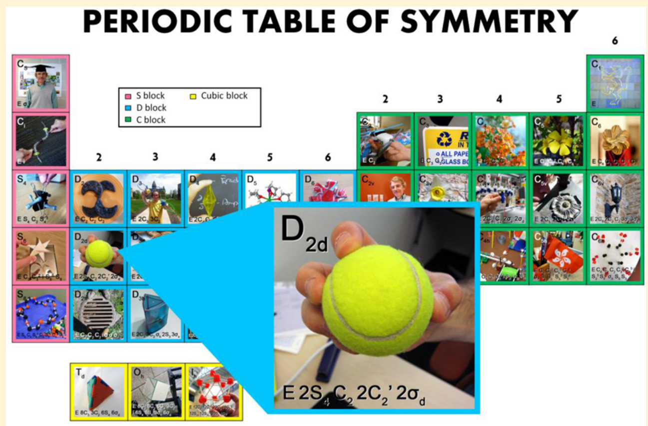
Ⓢ *Supporting Information*

ABSTRACT: A hands-on symmetry project is proposed as an innovative way of teaching point groups to undergraduate chemistry students. Traditionally, courses teaching symmetry require students to identify the point group of a given object. This project asks the reverse: students are instructed to identify an object that matches each point group. Doing so requires students to think about symmetry in their everyday environment and aids in the development of a more intrinsic understanding of the assignment of symmetry classifications.



KEYWORDS: *Second-Year Undergraduate, Inorganic Chemistry, Humor/Puzzles/Games, Hands-On Learning/Manipulatives, Group Theory/Symmetry, Collaborative/Cooperative Learning*

The conceptualization and integration of point group symmetry is a topic of much importance for undergraduates in chemistry. A strong foundation in symmetry provides students with a powerful tool for understanding advanced topics such as group theory, ligand design, spectroscopy, and crystallography. Therefore, effective teaching methods and activities for introducing symmetry are valuable tools for instructors. We propose a new type of project that asks students to identify objects from their surroundings that match specific point groups. This approach differs from typical textbook problems, which usually ask students to assign point groups to designated objects described or shown as pictures in text. This project requires the students to fully understand the symmetry operation(s) (i.e., rotational symmetry, mirror plane, etc.) of each point group and challenges them to carefully analyze and interact with more objects than the limited number provided in a textbook.

The final product of this project is a table reminiscent of the familiar periodic table of elements, but each element is replaced by a point group with a corresponding image of an object that the students found around their campus. The result of our investigation of objects on the St. Olaf College campus is shown in Figure 1.

■ CONCEPT

There are many methods for introducing symmetry and point groups to students. Some of these include online tools, software, and books with images from around the world to help visualize point groups.[1−5] In the conventional teaching of symmetry, students are prompted to assign a point group and/or symmetry operations to a given object or image.[6−20] Other, more hands-on approaches, such as those introduced by Flint and Scalfani et al., encourage students to understand and better visualize symmetry through 3D models.[21,22] Another valuable activity is provided on the Virtual Inorganic Pedagogical Electronic Resource (VIPEr) Web site, called the “Symmetry Scavenger Hunt.” This activity resembles our project in also having students seek various point groups around campus.[23]

By participating in our proposed activity, students grasp the shapes and patterns necessary to analyze molecules and objects in terms of group theory. Before looking for an object, the student is required to find and understand the symmetry operations that comprise each point group. The next task is to visualize the type of object that fulfills each of the symmetry operations. Some point groups are extremely difficult to find or imagine. Examples of lower order symmetry such as C_2 or C_{2h} may also prove rather difficult to find because candidates for such point groups may turn out on closer observation to have D_{2h} or other higher order point groups. Many objects have very similar sets of symmetry operations; therefore, ultimately the most challenging task is to find an object with certain symmetry operations while excluding others.

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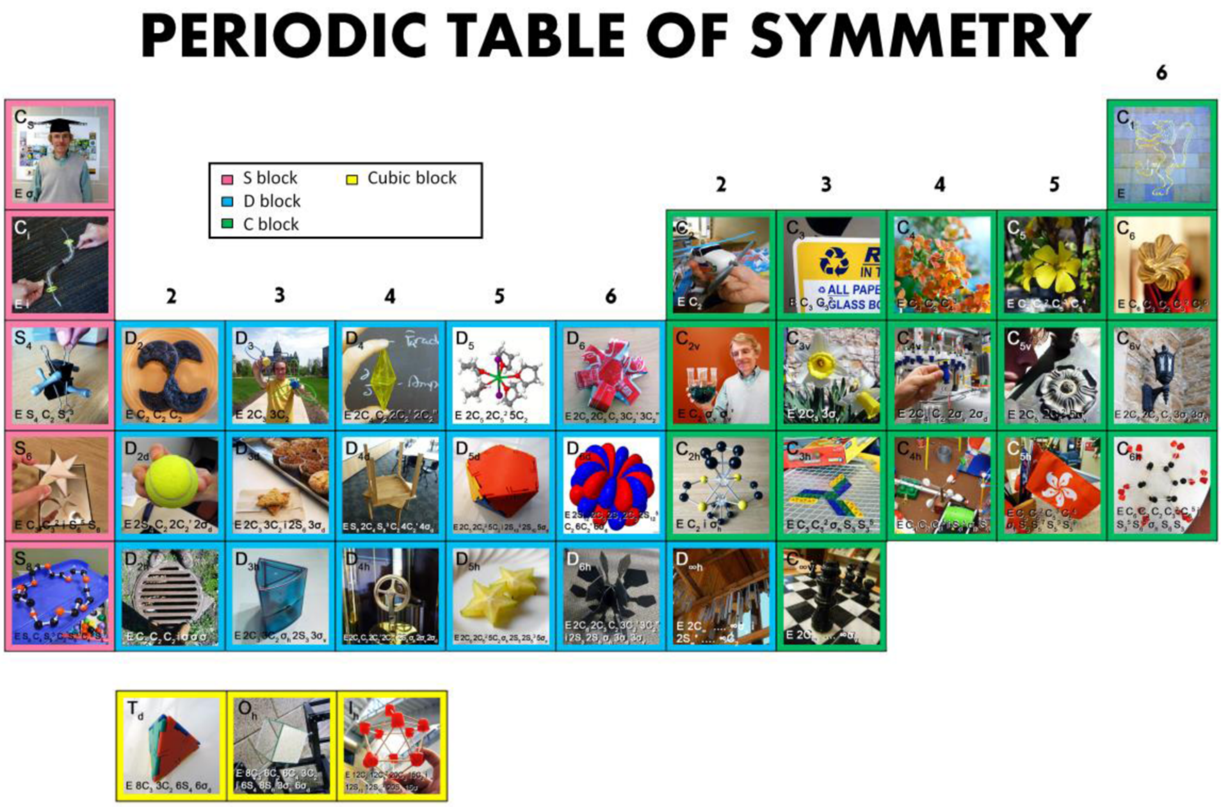

Figure 1. Periodic table of symmetry (larger image is provided in the Supporting Information).

To understand this project better, the following is a more detailed explanation of the student's thought process. For example, a student is assigned the point group D_{4d}. To find a suitable object, the student may first decompose the point group into its corresponding symmetry operations. Understanding the operations that are required to fulfill the assignment of this point group will be an important first step to visualizing the type of object that is needed. For example, in Figure 1 for the D_{4d} point group, we have an image of two, four-legged tables stacked such that the top table is flipped and turned 45° instead of perfectly mirroring the bottom table. For this object, the student must first understand that if there were only one table, it would fulfill the C_4 rotational symmetry operation of D_{4d}, but a single table would have no perpendicular C_2 axis of rotation required for the D point group assignment. The student can then further stack two tables so they are mirror images, but the pair would still have only D_{4h} symmetry; therefore the student must figure out a way to remove the horizontal plane of symmetry while adding the vertical planes of symmetry ($4\sigma_d$) of D_{4d}. Finally, once the student rotates the top table by 45°, the student will remove the horizontal plane of symmetry, and the pair of tables will fulfill the D_{4d} point group assignment.

In addition to finding an object matching a particular point group, this project requires the students to write a short explanation of how their object fulfills their assigned point group. For example, for D_{5d} and T_d symmetry, we have assembled toys with the correct shapes, but with faces of different colors; our explanation notes that the different colors on the faces should not be considered for their point group assignment. In many cases, the objects we have chosen, for example flowers, require simplification to fulfill the point group. This written portion will reveal to the instructor the extent of the student's understanding of the point group by explaining what is required, and what must be omitted in order to fulfill the assigned point group. A complete list of the explanations for each photo in our periodic table of symmetry is provided in the Supporting Information.

Traditional teaching methods give students a limited selection of objects and a specific number of point groups; however, this new project gives the students an open-ended point group challenge.[1,7−9] It is likely to be far more challenging for students to find point groups in objects when they have the whole world to work with, instead of a limited selection of objects. We hope that this project allows for the students to be more engaged and provides them another tool to learn symmetry from a different perspective.

■ DESIGN

In an effort to create a visualization displaying the symmetry in everyday objects, such as a flower, we captured pictures of objects observed on our campus with various symmetries. A photo for each point group was obtained and the collection of photos was organized into blocks. As a project connected to chemistry, adopting the form of a periodic table was opportune; however, to prevent confusion, the instructor should emphasize that there is no genuine connection between the point group table the students are constructing and the periodic table of elements. From the table, students can visualize the hierarchy of increasing point group symmetry and be reminded of the point group labels by associating the labels (S, D, or C) with the blocks in the original periodic table of elements. This table will not replace any teaching material provided in textbooks such as

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a point group flowchart, but instead, it can help provide some clarity to students for whom the current method is not intuitive.

A general guide to the table is provided in Figure 2: within the C and D blocks, reflection plane symmetry is designated by

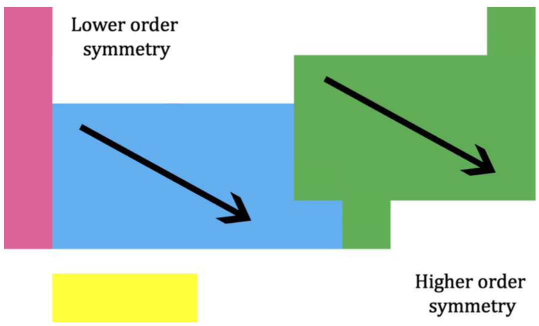

Figure 2. Higher order symmetry is observed at the bottom of the table. Intrablock symmetry is increased in groups to the right.

rows (i.e., the first row of the C/D block has no reflection plane symmetry, the second row has vertical or dihedral plane symmetry, and the third row has horizontal plane symmetry). However, in the C block the C_1 symmetry is ignored in this trend, and the order of rotational symmetry given by column increases toward the right in each block. Higher order symmetry is also found near the bottom in each block. The S block includes C_s and C_i, despite their labels because their operation is equivalent to S_1 and S_2, respectively. Much to our enjoyment, certain connections to the periodic table were kept such as the S and D blocks along with the general shape.

The photos in Figure 1 were taken primarily in the course of two days, with revisions made later. As constraints for our project, only pictures taken on campus were eligible and no pictures from the Internet were allowed. To make the project more creative, we made an attempt to allow as few molecular structures as possible. Molecular models and orbital calculations were allowed when no other option was present. Future projects could choose more or less strict interpretations of these conditions. The final product, after revision by faculty and students, was printed and displayed for all to view. A copy of the final document, in addition to a list of detailed explanation for each point group assignment, has been made available in the Supporting Information.

In our selection of each point group object, measures were taken to find objects that represented a definite symmetry. A tour of campus led to the discovery of the common point groups such as D_{2d}, D_{4d}, D_{4h}, $D_{\infty h}$, $C_{\infty v}$, C_1, C_2, C_4, and C_{4v}, in contrast to the less common higher-order symmetries such as O_h, C_5, C_{5v}, C_{5h}, and C_{6v}, which were more difficult to find. Further exploration and imaginative creations revealed objects displaying uncommon symmetries (D_2, D_6, D_{3d}, C_{6h}, S_4, and S_6). Computational modeling was allowed for two objects, D_5 and D_{6d}.

Some of the pictures in the table have difficult symmetry operations to identify. For instance, the bipyramidal structure of D_4 symmetry has dots along the corners of a face near the square plane of the shape. A fun addition to our table, C_{4h}, is a component of St. Olaf College's Rube Goldberg machine, which is a four-paddled propeller. As previously mentioned, a complete list of the explanations for each photo in our periodic table of symmetry is provided in the Supporting Information that will discuss these details.

■ CONCLUSION

The proposed Periodic Table of Symmetry project is designed to help chemistry students understand symmetry by finding everyday objects that correspond to given point groups. It will enhance students' understanding of how to apply all of the symmetry operations of a point group and emphasizes the challenging component of correctly including, and more importantly, excluding symmetry operations to ensure that the object has the correct point group. This project is recommended not as a replacement for traditional approaches of classifying molecules but rather as a project that will help enliven students' interest in symmetry as well as solidify their understanding after the basic symmetry principles have been explained.

■ ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available on the ACS Publications website at DOI: 10.1021/acs.jchemed.5b00325.

Instructor information. (PDF)
Instructor information. (DOCX)
Student handout. The student handout contains the instructions that will guide students through the activity. (PDF)
Student handout. The student handout contains the instructions that will guide students through the activity. (DOCX)
Example final project. In addition, a sample final project is provided for instructors' use. (PDF)
Example final project. In addition, a sample final project is provided for instructors' use. (DOCX)
Blank periodic table of symmetry. (PDF)
Blank periodic table of symmetry. (DOCX)

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Notes

The authors declare no competing financial interest.

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